

CS 200 Algorithms and Data Structures

Written Assignment #4

Due on Apr. 13 at the beginning of the class

[Problem 1] 5pts

List the 16 different relations on the set $\{0,1\}$.

[Problem 2] 5 pts (2+2+2+2+2)

For each of these relations on the set $\{1,2,3,4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

- (1) $\{(2,2),(2,3),(2,4),(3,2),(3,3),(3,4)\}$
- (2) $\{(1,1),(1,2),(2,1),(2,2),(3,3),(4,4)\}$
- (3) $\{(2,4),(4,2)\}$
- (4) $\{(1,2),(2,3),(3,4)\}$
- (5) $\{(1,1),(2,2),(3,3),(4,4)\}$

[Problem 3] 5 pts (1+1+1+2)

Let R be the relation on the set $\{1,2,3,4\}$ containing the ordered pairs $(1,1)$, $(1,2)$, $(1,3)$, $(2,3)$, and $(2,4)$. Find

- (1) R^2
- (2) R^3
- (3) R^4
- (4) R^5

[Problem 4] 5 pts

Let R be a reflexive relation. Show that R^n is reflexive for all positive integers n .
(Hint: use mathematical induction on n)

[Problem 5] 5 pts (2+1+1+1)

Represent each of these relations on $\{1,2,3,4\}$ with a **matrix**

- (1) $\{(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)\}$
- (2) $\{(1,1),(1,4),(2,2),(3,3),(4,1)\}$
- (3) $\{(1,2),(1,3),(1,4),(2,1),(2,3),(2,4),(3,1),(3,2),(3,4),(4,1),(4,2),(4,3)\}$
- (4) $\{(2,4),(3,1),(3,2),(3,4)\}$

[Problem 6] 5 pts (1+1+1+1+1)

How many **nonzero entries** does the matrix representing the relation R on $A = \{1,2,3,4,\dots,10\}$ consisting of the first 10 positive integers have in R is

- (1) $\{(a,b)|a \leq b\}$?
- (2) $\{(a,b)|a = b \pm 1\}$?
- (3) $\{(a,b)|a+b = 10\}$?
- (4) $\{(a,b)|a+b \leq 10\}$?
- (5) $\{(a,b)|a \neq 0\}$?