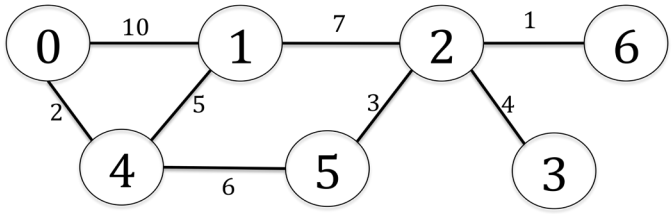
Worksheet for the Final Exam - Part I. Graphs

Date and Time: May 10 2012 Thursday 11:50AM~1:50PM
Location: Eng 120

Start with the Self-Test Exercises (pp.816) in Prichard.

1. Give the adjacency matrix and adjacency list for



2. Draw a graph modes, starting the type of graph used, to represent airline routs where every day there are four flights from Boston to Newark, two flights from Newark to Boston, three flights from Newark to Miami, two flights from Miami to Newark, one flight from Newark to Detroit, two flights from Detroit to Newark, three flights from Newark to Washington, two flights from Washington to Newark, and one flight from Washington to Miami, with

(1) an edge between vertices representing cities that have a flight between them (in either direction)

(2) an edge between vertices representing cities for each flight that operates between them)in either direction)

(3) an edge between vertices representing cities for each flight that operates between them (in either direction) plus a loop for a special sightseeing trip that takes off and lands in Miami

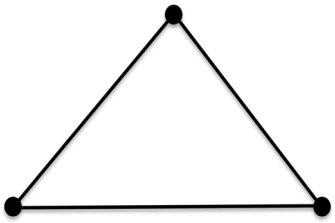
3. Describe a graph model that represents whether each person at a class knows the name of each other person at the class. Should the edges be directed or undirected? Should multiple edges be allowed? Should loops be allowed?

4. What do the in-degree and the out-degree of a vertex in a directed graph modeling a round-robin tournament represent?

5. Show that in a simple graph with at least two vertices there must be two vertices that have the same degree.

Graph A

6. How many subgraphs with at least one vertex does Graph A have?



Graph B

7. How many subgraphs with at least one vertex does Graph B have?

8. Let G be a graph with v vertices and e edges. Let M be the maximum degree of the vertices of G, and let m be the minimum degree of the vertices of G. Show that

(1) 2e/v >= m

(2) 2e/v <=M

9. Are the simple graphs with the following adjacency matrices isomorphic?

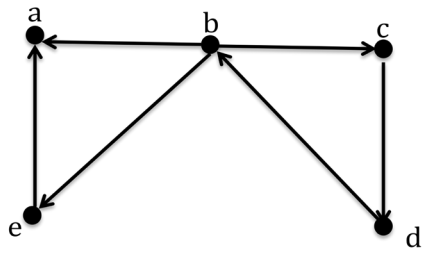
(1)

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

(2)

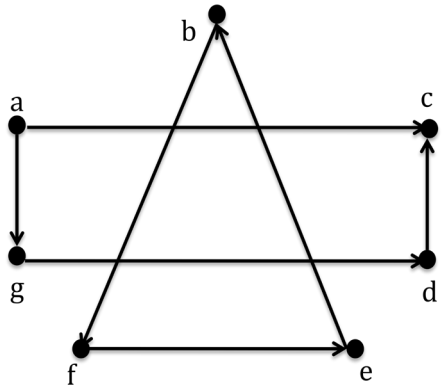
$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

10. Determine whether each of these graphs is strongly connected and if not, whether it is weakly connected.
(1)



Graph C

(2)



Graph D

11. Consider Figure A, and answer the following
    a. Will the adjacency matrix be symmetrical?
    b. Provide the adjacency matrix
    c. What does the sum of each row of the adjacency matrix represent?
    d. Provide the adjacency list

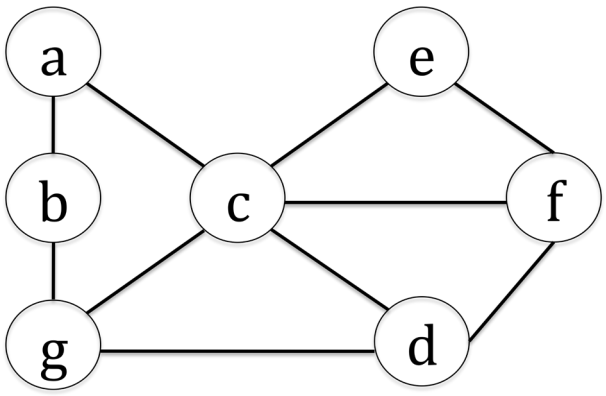


Figure A

12. Describe an adjacency matrix for a complete graph.

13. Do any complete graphs have Euler circuits? If so, describe the characteristics of such graphs.

14. Do any complete graphs have Hamiltonian circuits? If so, describe the characteristics of such graphs.

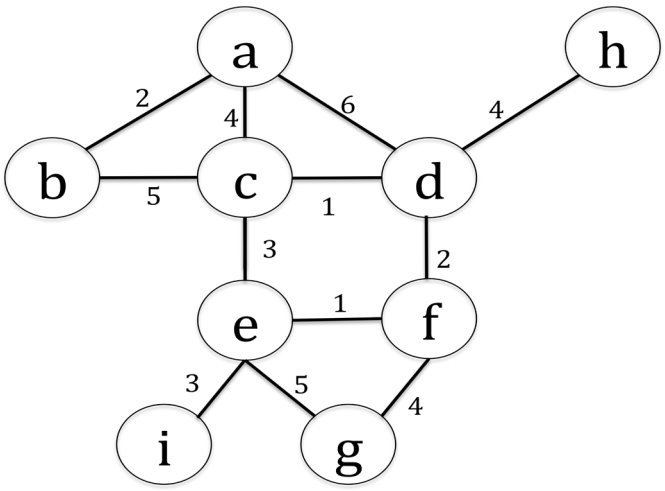


Figure X

15. For the Figure X, use DFS to traverse the graph beginning with vertex b.

16. For the Figure X, use BFS to traverse the graph beginning with the vertex b.

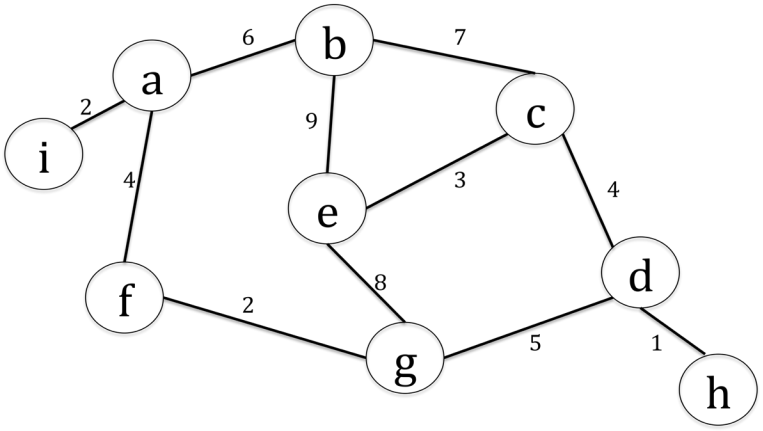


Figure Y

17. Draw the minimum spanning tree for the graph in Figure Y when you start with vertex e.

18. Draw the minimum spanning tree for the graph in Figure Y when you start with vertex d.

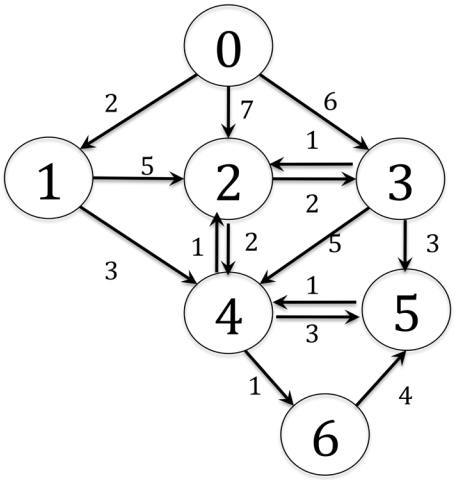


Figure Z

19. Trace the shortest-path algorithm for the graph in Figure Z, letting vertex 0 be the origin

20. Determine a Euler circuit for the graph in Figure W. Why is one possible?

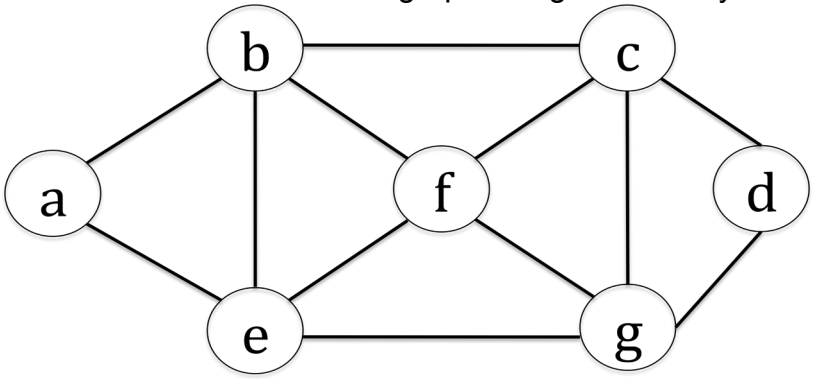


Figure W

21. Prove that a connected undirected graph with n vertices and more than n-1 edges must contain at least one simple cycle.

22. Prove that a graph-traversal algorithm visits every vertex in the graph if and only if the graph is connected, regardless of where the traversal starts.

23. How many edges are there in a graph with 20 vertices each of degree 10?
2 * e = 20*10 e= 100

24. Can someone cross all the bridges shown in this map (Figure F) exactly once and return to the starting point?

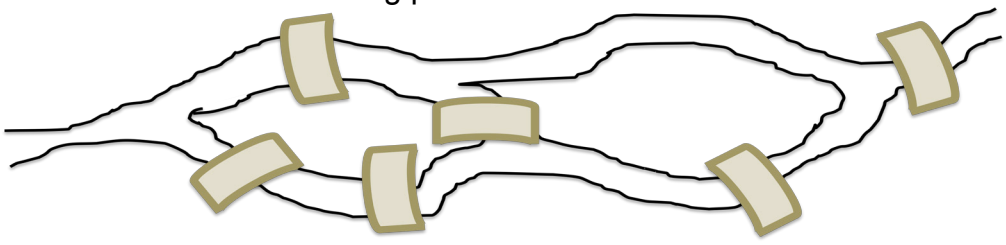


Figure F

25. Is Graph in Figure Q, planar?

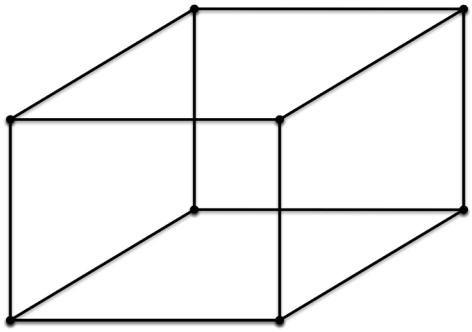


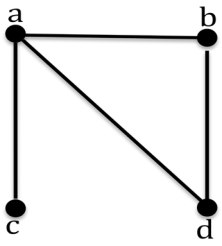
Figure Q

26. Suppose that a connected planar graph has 30 edges. If a planar representation of this graph divides the plane into 20 regions, how many vertices does this graph have?

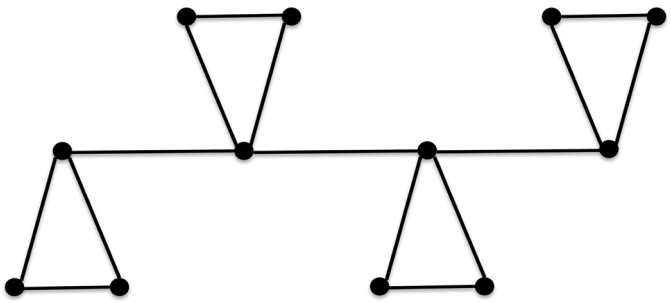
27. Can five houses be connected to two utilities without connections crossing?

28. Which graphs have a chromatic number of 1?

29. Find the chromatic number of the given graph H and I.



Graph H



Graph I