More About Induction

Recitation 2

Example - Induction in excruciating detail

I will admit to getting this example from Wikipedia [1], but it is a good place to start. The following is the formula, called p(n), for the summation of natural numbers:

$$0 + 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

Basis step: Show that it holds for n = 0.

$$0 = \frac{0 \cdot (0+1)}{2}$$
$$0 = \frac{0}{2} = 0$$

- Start with only 0 on the left side, and n = 0 on the right side of the previous equation. Do the arithmetic, and if the left side equals the right side, you have proved the basis. Or shown that p(0) holds, depending on your terminology.

Inductive Step: We now need to show that if p(k) holds, then p(k+1) also holds. See later note about n and k. *Assume that p(k) holds, for some unspecified value of k.* What we want to show is that p(k+1) holds:

$$(0 + 1 + 2 + \cdots + k) + (k+1) = \frac{(k+1)((k+1)+1)}{2}$$

- Notice what was done here. The left side started out the same as the left side of the very first equation, and then had a '(k+1)' added to it. The right side started out the same as the right side of the very first equation. Then, every k was robotically replaced with a '(k+1)'.

We already have a formula for p(k), from the original equation. We can substitute that for the '(0+1+2+...+k) term above:

$$\frac{k(k+1)}{2} + (k+1) = \frac{(k+1)((k+1)+1)}{2}$$

What we must now do is make the left side match the right side.

$$\frac{k(k+1) + 2(k+1)}{2} = \frac{(k+1)((k+1)+1)}{2}$$

$$\frac{k^2+3k+2}{2} = \frac{(k+1)((k+1)+1)}{2}$$
$$\frac{(k+1)(k+2)}{2} = \frac{(k+1)((k+1)+1)}{2}$$
$$\frac{(k+1)((k+1)+1)}{2} = \frac{(k+1)((k+1)+1)}{2}$$

Left now equals right, we are done. We have shown the p(k+1) holds by showing that the left and right sides of the original equation are still equivalent in the (k+1) case just like they were for k.

About k and n, the notation changes depending on which part of the problem is being done. n is usually used for describing the original equation. k is used during the algebra for the inductive step, but it is the same variable renamed. I think this is to create visual distinction between the original equation and the later algebra work. For our purposes n == k.

Exercise for today

- Is a work sheet with more induction practice. This is also your attendance sheet, show it to me when you are done or at the end of the period and I will record the attendance. You may keep the worksheet if you want to.
- I will post the answers on the projector towards the end of the period.

Optional extra problem

If you are done with the worksheet with time to spare, and want to do some programming, try this problem. It's not worth any points, but is good practice:

- Look up the Lucas Numbers on Wikipedia
  - http://en.wikipedia.org/wiki/Lucas_number
- This is a series of numbers similar to the Fibonacci series.
- Make a new eclipse project, and write a recursive function which computes the n^{th} Lucas Number.
- This description is intentionally vague

References

[1] Wikipedia. Mathematical induction — wikipedia, the free encyclopedia. `http://en.wikipedia.org/w/index.php?title=Mathematical_induction&oldid=%472683752`, 2012. [Online; accessed 29-January-2012].