EXAM 2 REVEIW

CS200 RECITATION 9

Trees

Hierarchical Abstract Data Type (ADT)

- Root node at the top
  - All other nodes descend from the root
- May be either **position oriented** or **value oriented**
  - position oriented: ADT primarily cares about an element's position. e.g. arrays, lists, stacks...
  - value oriented: ADT primarily cares about an element's value. e.g. anything sorted, binary search trees...
- Terminology:
  - node: basic element of the tree
  - parent of node n: node directly above n in the tree
  - child of node n: node directly below n in the tree
  - root: start of tree, only node with no parent
  - leaf: node with no children
  - edge: connection between two nodes
  - Prichard pp. 569 has a nice list of terms
- Traversals, orders for moving through all the nodes
  - in-order:
    * "left, root, right"
    * if done to a BST or similar, result will be in sorted search-key order
  - pre-order:
    * "root, left, right"
    * commonly used to parse arithmetic / algebra expressions (2+2 -> + 2 2)
  - post-order:
    * "left, right, root"
    * also commonly used to parse arithmetic / algebra
- Two ways to represent when programming:
  - reference based: each node has references to other node objects which are it's children. Default representation for this course (see previous labs)
  - array based: nodes are stored in an array, each node has indices into that array which are it's children

Figure 1: Trees in Computer Science have a confusing relationship with gravity. Image credit: By Borealis55 17:36, 27 November 2007 (UTC) (Own work) [Public domain], via Wikimedia Commons

Binary Search Tree

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Is a Binary Tree (every node has at most 2 children), such that:

- any node's left subtree only contains nodes with keys less than the current node
- any node's right subtree only contains nodes with keys greater than current node

Basic properties:

- Full: tree of height h with no missing nodes, all leaves are at level h and all other nodes have two children
- Complete: tree of height h which is full to level h - 1, level h filled left to right
- Balanced: left and right subtrees of any node have height which only differs by 1.

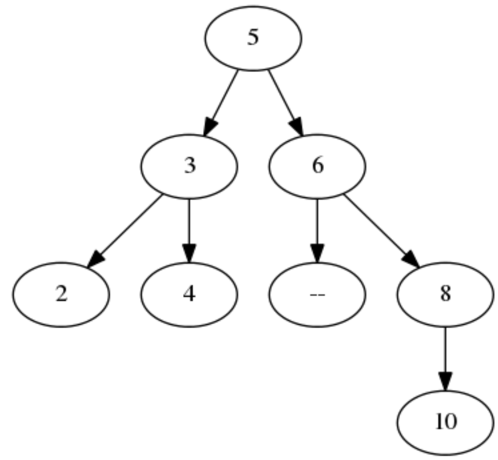


Figure 2: a BST

Operations:

- Search:
  - move through tree looking for a search key. At any node, compare search key with that node's data. If less than, go left. If greater than, go right. If equal, search key found.
- Insert:
  - move through tree in same manner as search, looking for an empty child node to put the data in
- Delete, depends on type of node
  - if it's a leaf node, just delete it
  - if it has only one child, delete it and replace with child
  - if it has two children (is in middle of tree): find the in-order successor node, swap this node's data with the successor, delete the successor.

Tree sort: put all data into a BST, do an in-order traversal. The tree's nodes will be visited in sorted search key order.
Big O complexity: Search, insert, delete all have same performance or O(log n) average and O(n) worst case. Traversals is always O(n).

2-3 trees

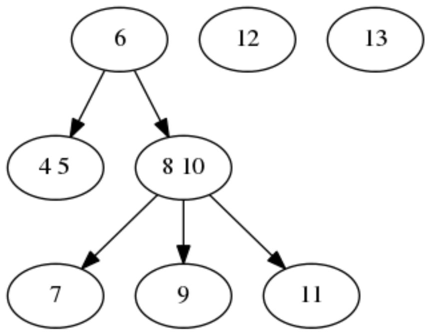
Improvement upon binary trees, designed to keep tree balanced when inserting and deleting data. Has two types of node:

- 2-node: one data element, 2 child nodes
- 3-node: two data elements, 3 child nodes

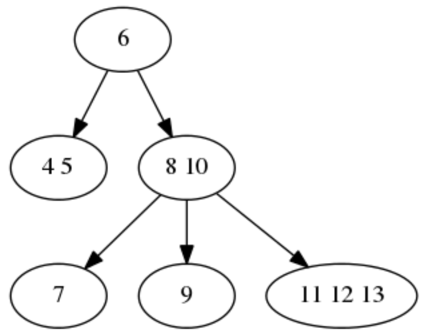
2-3 trees retain the left < root < right ordering of binary search trees, thus algorithms such as search and traversal are very similar to those in BSTs. Only new thing is 3-nodes, which have two data elements, recorded in sorted order. When visiting a 3-node in traversal / search, both it's elements are visited or searched in order before moving on to the next node.

- Insert is a bit different:
  - Locate the leaf at which a search for the data to be inserted would terminate.
  - Add the data to that leaf, one of two things happens:
    - A) leaf contains two or less data items: done!
    - B) leaf contains three data items: must split leaf. if leaf was (low, mid, high):
      - promote mid to the parent node (move it up there)
      - low and high become 2-nodes under the parent node
      - if promotion of mid over filled the parent, recurse to split the parent in the same manner
      - if splitting the root, mid becomes the new root as a 2-node
- Delete is similar:
  - Locate data to delete:
    - if it is a leaf node, just delete it.
    - if it is not a leaf, swap data with in-order successor. Delete the successor (which will be a leaf)
      - this may cause an empty node somewhere down the tree: if a sibling has two items: rotate into the empty node if no sibling has two items: combine child and parent into a 3-node

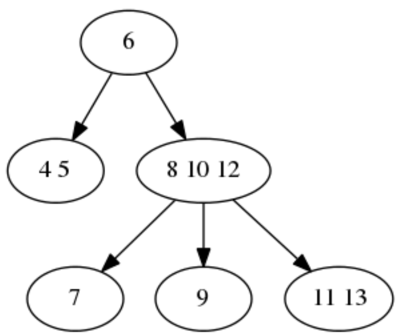
An example of inserting (on next page). **Note that these are not necessarily correct 2-3 trees, they are purely an example of insertion.** Also, the 4-node is purely for explanation, since several of these steps probably happen within the same method call.



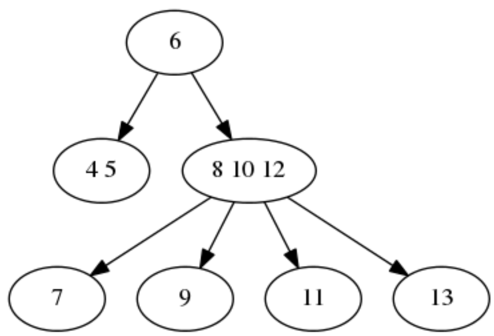
(a) Want to insert 12 and 13



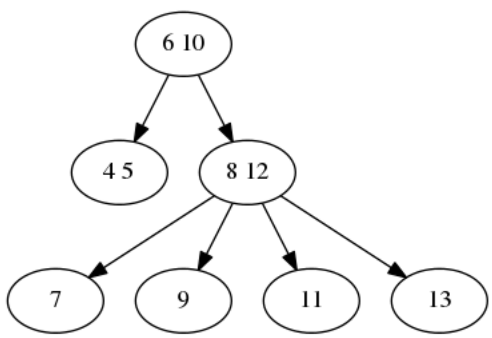
(b) Both got added to same node, which is now over filled



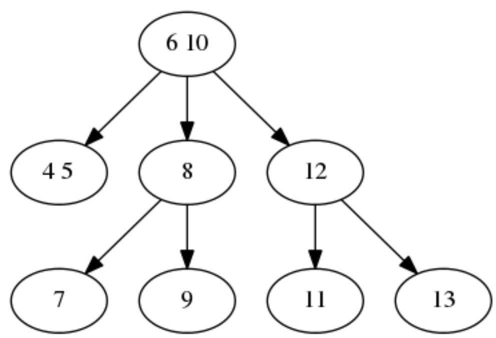
(c) Promote 12 (middle one) to parent node, overfilling it



(d) Reassign chilren, so they follow the search tree rules



(e) Promote 10 again into it's parent (happens to be the root)



(f) 3-node can't have 4 children, and it does not follow the search tree rules. Break up node (8, 12) and reassign.

Figure 3: Example of inserting stuff into a 2-3 tree. Note, these 2-3 tree are not necessarily correct, purely an example of insert

2-3-4 tree

Improvement upon the 2-3 tree. many algorithms are very similar (search, traverse, etc...). Big difference is:

- has 4-nodes. Three data elements, four children.
- Insert: splits up any 4-nodes it encounters while looking for insertion point
  - then inserts into whichever node make sense, this may generate a 4-node which will be split up late when another insert finds it

  - to split a 4-node: move it's middle data element into the parent node. if the parent is now a 4-node, it will be split by the next insert.

- Delete:
  - find the node
  - swap with in-order successor (always delete in a leaf)
  - if leaf is a 3 or 4-node, remove item
  - if you never delete from a 2-node, deletion can be done with only one pass through tree (transform each 2-node into a 3 or 4-node while searching for delete item)

Grading

Attendance only, come sign sheet