Part 10. Graphs

CS 200 Algorithms and Data Structures

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Outline

- Introduction
- Terminology
- Implementing Graphs
- Graph Traversals
- Topological Sorting
- Shortest Paths
- Spanning Trees
- Minimum Spanning Trees
- **Circuits**

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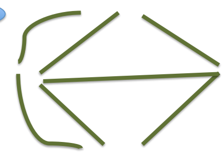
Circuits

- Cycle
  - A special cycle that passes through every vertex (or edge) in a graph exactly once and returns back to the place it started.

Euler's bridge problem (Bridges of Konigsberg Problem)



Euler

Is it possible to travel across every bridge without crossing any bridge more than once?

http://yeskarthi.wordpress.com/2006/07/31/euler-and-the-bridges-of-konigsberg

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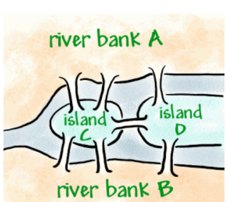
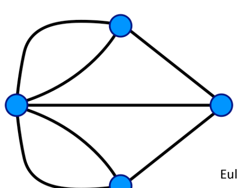
Euler's bridge problem (Bridges of Konigsberg Problem)





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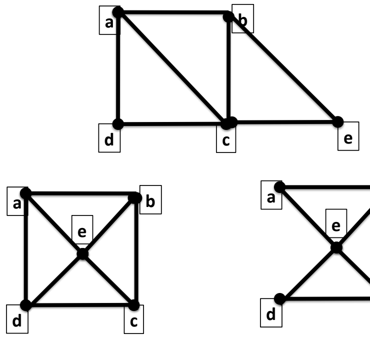
Euler paths/circuits

- Euler path: A path that visits each edge only once in the graph
- Euler circuit: A cycle that visits each edge only once in the graph

Example: Does any graph have an Euler circuit?



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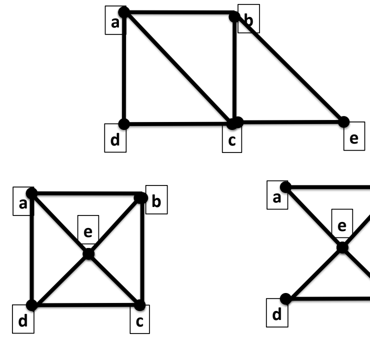
Example: Does any graph have an Euler path?



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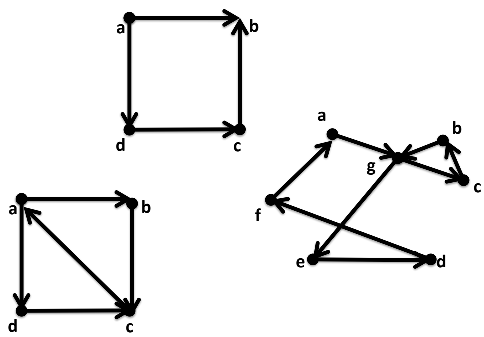
Example: Does any graph have an Euler circuit?



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Euler paths/circuits

- Is there a simple criterion that allows us to determine whether a graph has an Euler circuit or path?

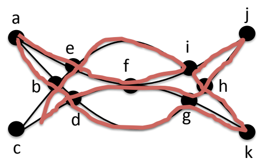
Euler Paths

- Theorem: A connected multigraph has an Euler path iff it has exactly two vertices of odd degree

Euler Circuits

- Theorem: A connected multigraph with at least two vertices has an Euler circuit iff each vertex has an even degree.

Mohammed's Scimitars



Can Mohammed's scimitars be drawn without lifting a pencil and the drawing begins and ends at the same point?

a-e-f-i-e-b-c-d-g-h-j-i-k-g-d-b-a

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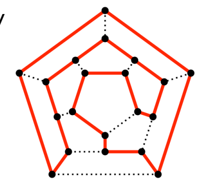
Hamiltonian Paths/Circuits

- A Hamiltonian path/circuit: path/circuit that visits every **vertex** exactly once.
- Defined for directed and undirected graphs.
- Is there an efficient way to determine whether a graph has a Hamiltonian circuit?



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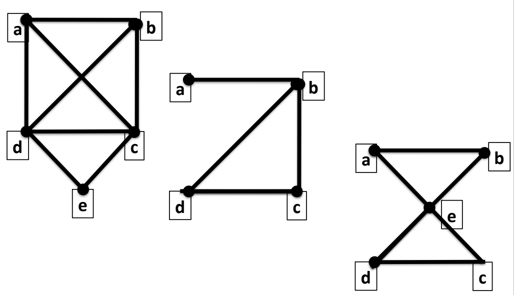
Does any graph have a Hamilton circuit or a Hamilton path?



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DIRAC's Theorem

- If G is a simple graph with *n* vertices with $n \geq 3$ such that the degree of every vertex in G is at least n/2, then G has a Hamilton circuit.

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Ore's Theorem

- If G is a simple graph with n vertices with $n \geq 3$ such that $deg(u)+deg(v) \geq n$ for every pair of nonadjacent vertices u and v in G, then G has a Hamilton circuit.

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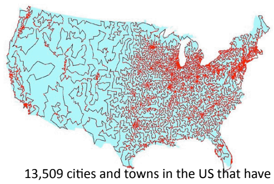
Hamiltonian Paths/Circuits

- Direc and Ore's theorems provide sufficient condition for a connected simple graph to have a Hamilton circuit.
  - They do NOT provide necessary condition for the existence of a Hamilton circuit
- This problem belongs to a class of problems for which it is believed there is no efficient (polynomial running time) algorithm.

The Traveling Salesman Problem (TSP)

TSP: Given a list of cities and their pairwise distances, find a shortest possible tour that visits each city exactly once.

An optimal TSP tour through Germany's 15 largest cities (one out of 14!/2)



13,509 cities and towns in the US that have more than 500 residents

http://www.tsp.gatech.edu/

Using Hamilton Circuits

- Examine all possible Hamilton circuits and select one of minimum total length
- With n cities..
  - (n-1)! Different Hamilton circuits
  - Ignore the reverse ordered circuits
  - (n-1)!/2
- With 50 cities
- 12,413,915,592,536,072,670,862,289,047,373,375,038,521,486,354,677,760,000,000,000 routes

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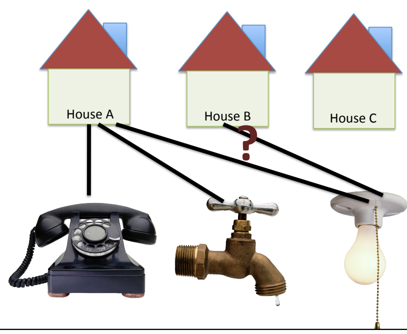
The three utilities problem



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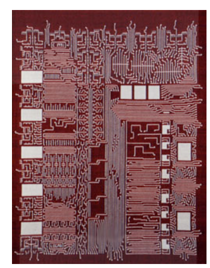
Planar Graphs

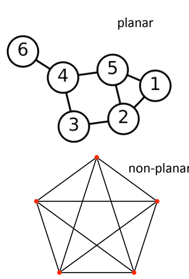
- You are designing a microchip – connections between any two units cannot cross



http://www.dmoma.org/

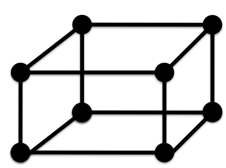
Planar Graphs

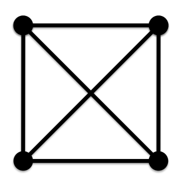
- You are designing a microchip – connections between any two units cannot cross
- The graph describing the chip must be planar



http://en.wikipedia.org/wiki/Planar_graph

Is this graph planar?





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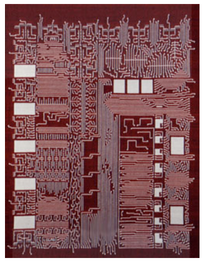
Chip Design

- You want more than planarity: the lengths of the connections need to be as short as possible (faster, and less heat is generated)



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