Part 7. Tables and Priority Queues

CS 200 Algorithms and Data Structures

1

Outline

- **Tables**
- Priority Queues
- Heaps
- Heapsort

2

Value Oriented Data Structures

- Value-oriented operations are very common:
  – Find John Smith's facebook
  – Retrieve the student transcript of Ruth Mui
  – Register Jack Smith for an Amazon Cloud account
  – Add a user to a database (e.g. Netflix database).
- To support such uses: Arrange the data to facilitate search/insertion/deletion of an item given its search key

3

Example: Table of Student Points

Student ID	Student First Name	Student Last Name	Exam 1	Exam 2
001245	Jake	Glen	67	89
001247	Parastoo	Mahgreb	87	78
001256	Wayne	Dwyer	90	96
012345	Bob	Harding	45	50
022356	Mary	Laing	95	97

•Each row is a record
•Each column is a field
•Student ID is the search key field

Search Keys

- In many applications the search key must be unique to a record
  – It identifies a *single* record
  – Records with the same search key must not exist in these tables
- Records should be arranged to facilitate search for an item using the search key field
- The search key of a record must not change while it is in the table. Why?

Table ADT

- Operations
  – Create empty
  – Is empty?
  – Size
  – Insert new item
  – Delete item with search key
  – Retrieve item by search key
  – Traverse items
    - Sorted or unsorted? Answer determines how the table is implemented
    - We focus on tables with sorted records (items)

Mapping Data Structures

7

Pseudocode for Table ADT

```
createTable() // creates an empty table
tableIsEmpty():boolean
  // Determines whether a table is empty
tableLength():integer
  // Determines the number of items (records) in a
   // table
tableTraverse():TableItemType
   // Traverses a table (in sorted search key order).
```

Pseudocode for Table ADT

```
tableInsert(in newItem:TableItemType) throws TableException
  //Inserts new record (newItem) into a table whose
  // items have distinct search keys that differ from record's
  // search key.
  // throws exception if unsuccessful.
tableDelete(in searchKey:KeyType): boolean
  // Deletes from a table the record whose search key equals
  // searchKey. Returns true if successful, false if no item found.
tableRetrieve(in searchKey:KeyType):TableItemType
  // Returns item whose search key equals searchKey.
  // Returns null if not there.
```

Table Records

```
public abstract class KeyedItem<KT extends
   Comparable <? super KT>>
//KT is constrained to be a type that implements comparable or is a
//subclass of a type which does so
{
   private KT searchKey;

   public KeyedItem(KT key) {
    searchKey = key; }//constructor

   public KT getKey() {
    return searchKey; }//accessor
}
```

There is no method to set the search key. Why?

Table Record Example

```
public class User extends KeyedItem<String> {
   private String StudentID;     // search key
   private String firstName;
   private String lastName; …

   public User(String userID, String _firstName, …)  {
    super(userID);//Why is super used here?
    firstName = _firstName; …
   }//constructor
```

Table Interface

```
public interface TableInterface<T extends KeyedItem<KT>,
            KT extends Comparable <? super KT>> {
   // Precondition for all operations:
   // No two items of the table have the same search key.
   // The table's items are sorted by search key (actually
not required)

   public boolean tableIsEmpty();
   // Determines whether a table is empty.
   // Postcondition: Returns true if the table is empty;
   // false otherwise

   public int tableLength();
   // Determines the length of a table.
   // Postcondition: Returns the number of items in the
table.
```

Table Interface (cont.)

```
public void tableInsert(T newItem)  throws TableException;
  // Inserts a record into a table in its proper sorted order according
  // to the item's search key.
  // Precondition: The record's (newItem's) search key must be
  // unique in the table.
  // Postcondition: If successful, newItem is in its proper order in
  // table. Otherwise, table is unchanged; throw TableException.

public boolean tableDelete(KT searchKey);
  // Deletes a record with search key KT from table.
  // Precondition: searchKey is the search key of item to be deleted.
  // Postcondition: If there is a record with KT in the table, the
  // item was deleted and method returns true. Otherwise, table is
  // unchanged; return false.
```

Table Interface (cont.)

```
public KeyedItem tableRetrieve(KT searchKey);
   // Retrieves a record with a search key KT from table.
   // Precondition: searchKey is search key for record to be retrieved.
   // Postcondition: If the retrieval was successful,
   // table record with search key matching KT is returned.
   // If no such record exists, return null.

}  // end TableInterface
```

Possible Implementations

- ArrayList sorted by search key
  - **Pros: fast access via binary search**
  - **Cons: may waste space; add and delete are expensive**
- Array sorted by search key
  - Similar to ArrayList
- Linked List sorted by search key
  - **Cons: expensive retrieval, add and delete**
- Binary search tree
  - **Pros: fast average access, efficient use of space**
  - **Cons: poor worst case performance**

Performance of Table Implementations

	Search	Add	Remove
Sorted array-based	O(log n)	O(n)	O(n)
Unsorted array-based	O(n)	O(1)	O(n)
BST*	O(log n) (O(n))	O(log n) (O(n))	O(log n) (O(n))

* Worst case behavior in parentheses

Outline

- Tables
- **Priority Queues**
- Heaps
- Heapsort

17

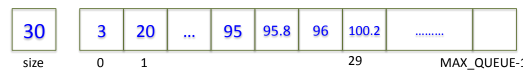
Priority Queues

- Characteristics
  - Items are associated with a priority
  - Provide access to one element at a time - the one with the highest priority
- Uses
  - Operating systems
  - Network management
    - Real time traffic usually gets highest priority when bandwidth is limited

Priority Queue ADT Operations

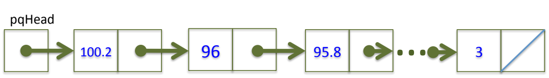
1. Create an empty priority queue
   `createPQueue()`
2. Determine whether empty
   `pqIsEmpty():boolean`
3. Insert new item
   `pqInsert(in newItem:PQItemType) throws PQueueException`
4. Retrieve and delete the item with the highest priority
   `pqDelete():PQItemType`

Priority Queue – Implementation (1/3)



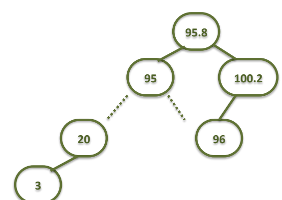
- ArrayList ordered by priority
  - Sorted ArrayList
  - Find the correct position for the insertion
  - Shift the array elements to make room for the new item

Priority Queue – Implementation (2/3)



- Reference-based implementation
  - Sorted in descending order
    - Highest priority value is at the beginning of the linked list
    - pqDelete returns the item that psHead references and changes pqHead to reference the next item.
    - pqInsert must traverse the list to find the correct position for insertion.

Priority Queue – Implementation (3/3)



- Binary search tree
  - What is the highest value of the nodes?
  - Removing is easy
    - It has at most one child
  - You can use a balanced variation of the binary search tree.
- Other options?

Outline

- Tables
- Priority Queues
- **Heaps**
- Heapsort

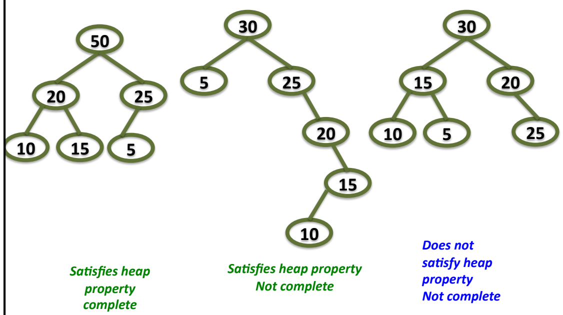
23

Heap - Definition

- A maximum heap (maxheap) is a ***complete* binary tree** that satisfies the following:
  - It is an empty tree

  or it has the heap property:
  - Its root contains a key greater or equal to the keys of its children
  - Its left and right subtrees are also heaps
- Implications of the heap property:
  - The root holds the maximum value (global property)
  - Values in descending order on every path from root to leaf
- **Heap is NOT a binary search tree.**

Examples

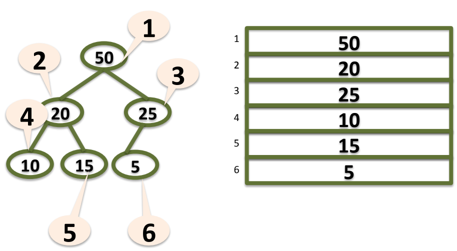


25

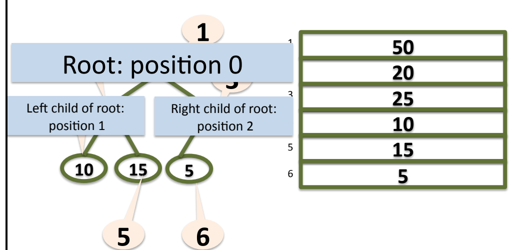
Heap ADT

```
createHeap()  // create empty heap

heapIsEmpty():boolean
   // determines if empty

heapInsert(in newItem:HeapItemType) throws
  HeapException
   // inserts newItem based on its search key. Throws
   // exception if heap is full

heapDelete():HeapItemType
   // retrieves and then deletes heap's root item
   // which has largest search key
```

ArrayList-based Implementation(1/3)



ArrayList-based Implementation (2/3)



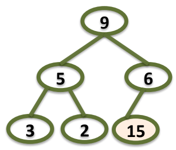
ArrayList-based Implementation (3/3)

- Traversal items:
  - Root at position 0
  - Left child of position i at position 2i+1
  - Right child of position i at position 2(i+1)
  - Parent of position i at position ⌊(i-1)/2⌋

Heap Operations - `heapInsert`

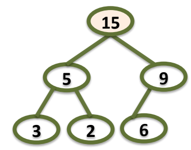
- **Step 1**: put a new value into first open position (maintaining completeness)
- **Step 2**: percolate values up
  - **Re-enforcing the heap property**
  - Swap with parent until in the right place

Insertion into a heap (Insert 15)



Insert 15
Trickle up
Trickle up

Insertion into a heap (Insert 15)



Heap Insert Pseudocode

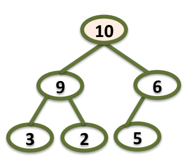
```
 // insert newItem into bottom of tree
items[size] = newItem
// percolate new item up to appropriate spot
place = size
parent = (place-1)/2
while (parent >= 0 and items[place] > items[parent])
  {
  swap items[place] and items[parent]
  place = parent
  parent = (place-1)/2
  }
increment size
```

Part of the insert operation is often called siftUp

Heap operations – `heapDelete`

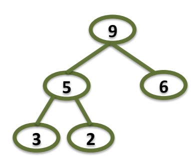
- **Step 1**: always remove value at root (Why?)
- **Step 2**: substitute with rightmost leaf of bottom level (Why?)
- **Step 3**: percolate/sift down
  - Swap with maximum child as necessary

Deletion from a heap



Delete 10
Place last node in root
Trickle down

Deletion from a heap



`heapDelete` Pseudocode

```
// return the item in root
rootItem = items[0]
//copy item from last node into root
items[0] = items[size-1]
size--
// restore the heap property
heapRebuild(items, 0, size)

return rootItem
```

`heapRebuild` Pseudocode

```
heapRebuild(inout items:ArrayType, in root:integer,
            in size:integer)
  if (root is not a leaf) {
    child = 2 * root + 1   // left child
    if (root has right child) {
      rightChild = child + 1
      if (items[rightChild].getKey() >
              items[child].getKey()) {
         child = rightChild
      }
    } // larger child
    if (items[root].getKey() < items[child].getKey()) {
      swap items[root] and items[child]
      heapRebuild(items, child, size)
  }
}
```

heapRebuild is also called siftDown

Array-based Heaps: Complexity

	Average	Worst Case
insert	O(log n)	O(log n)
delete	O(log n)	O(log n)

Heap versus BST for PriorityQueue

- BST can also be used to implement a priority queue
- How does worst case complexity compare?
- How does average case complexity compare?
- What if you know the maximum needed size for the PriorityQueue?

Small number of priorities

- A heap of queues with a queue for each priority value.

Outline

- Tables
- Priority Queues
- Heaps
- **Heapsort**

42

HeapSort

- Algorithm
  - Insert all elements (one at a time) to a heap
  - Iteratively delete them
    - Removes minimum/maximum value at each step
- Computational complexity?

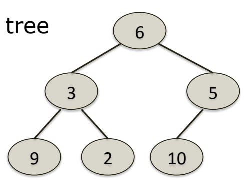
HeapSort

- Alternative method (in-place):
  - Create a heap out of the input array:
    - Consider the input array as a complete binary tree
    - Create a heap by iteratively expanding the portion of the tree that is a heap
      - Start from the leaves, which are semi-heaps
      - Move up to next level calling heapRebuild with each parent
  - Iteratively swap the root item with last item in unsorted portion and rebuild

Build initial tree

6	3	5	9	2	10

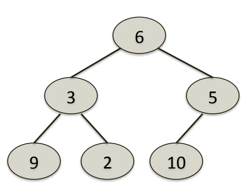
- Begin with the root
- Left to right down this tree



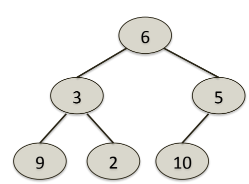
Transform tree into a heap

```
for (index = n -1 down to 0)
    //Assertion: the tree rooted at index is a semiheap
    heapRebuild(anArray, index x)
    //Assertion: the tree rooted at index is a heal
```

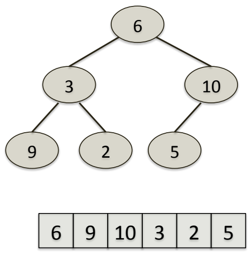
- Call heapRebuild() on the leaves from right to left
- Move up the tree



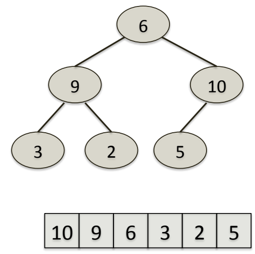
6	3	5	9	2	10



6	3	10	9	2	5



49



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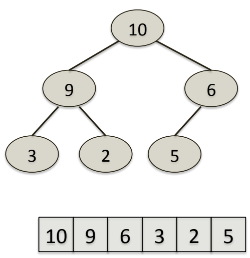
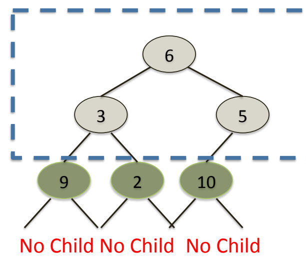


51

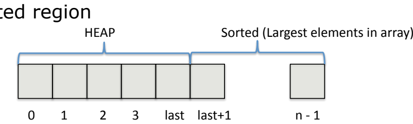
Do we need n steps?



52

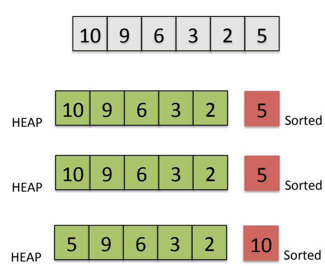
After transforming the array into a heap

- Heapsort partitions the array into two regions
  - Heap region
  - sorted region



53

Sorted : n-1 (5~)



54

Sorted: n-2 (4~)

10 | 9 | 6 | 3 | 2 | 5

9 | 5 | 6 | 3 | 2   10

HEAP 9 | 5 | 6 | 3   2 | 10 Sorted

2 | 5 | 6 | 3   9 | 10

55

Sorted: n-3 (3~)

10 | 9 | 6 | 3 | 2 | 5

6 | 5 | 2 | 3   9 | 10

HEAP 6 | 5 | 2   3 | 9 | 10 Sorted

HEAP 3 | 5 | 2   6 | 9 | 10 Sorted

56

Sorted: n-4 (2~)

10 | 9 | 6 | 3 | 2 | 5

HEAP 5 | 3 | 2   6 | 9 | 10 Sorted

HEAP 5 | 3   2 | 6 | 9 | 10 Sorted

HEAP 2 | 3   5 | 6 | 9 | 10 Sorted

57

Sorted: n-5 (1~)

10 | 9 | 6 | 3 | 2 | 5

HEAP 3 | 2   5 | 6 | 9 | 10 Sorted

HEAP 3   2 | 5 | 6 | 9 | 10 Sorted

HEAP 2   3 | 5 | 6 | 9 | 10 Sorted

58

Sorted :n-6 (0~)

10 | 9 | 6 | 3 | 2 | 5

HEAP 2   3 | 5 | 6 | 9 | 10 Sorted

2 | 3 | 5 | 6 | 9 | 10 Sorted

2 | 3 | 5 | 6 | 9 | 10

Final result

59

HeapSort Pseudocode

```
heapSort(ourItems:ArrayList, n:integer)
   // First step: build heap out of the input
   array
   for (index = n – 1 down to 0) {
    // Invariant: the tree rooted at index is a
   // semiheap
   // semiheap: tree where the subtrees of the
   //root are heaps
    heapRebuild(ourItems, index, n)
    // The tree rooted at index is a heap
   }
```

HeapSort Pseudocode

```
heapSort(ourItems:ArrayList, n:integer)
   for (index = n-1 down to 0) {
    heapRebuild(ourItems, index, n)
   }

   last = n -1 // initialize the regions
   for (step = 1 through n) {
    swap ourItems[0] and ourItems[last]
    decrement last
    heapRebuild(ourItems, 0, last)  }
```