MIMO Robust Control for Heating, Ventilating and Air Conditioning (HVAC) Systems

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Abstract—Potential improvements in heating, ventilating, and air conditioning (HVAC) system performance are investigated through the application of multiple-input, multiple-output (MIMO) robust controllers. This approach differs dramatically from today's prevalent method of building HVAC controllers using multiple single-input, single-output (SISO) control loops.

A simulation model of an experimental HVAC system is used in the design and simulation testing of controllers. While simulation can be insightful, the only way to truly verify the performance provided by different HVAC controller designs is by actually using them to control an HVAC system. Thus, an experimental system for testing advanced HVAC controllers is built. The construction and modeling of this system is the focus of a separate article [2].

While this system is only a portion of an overall HVAC system, it is representative of a typical hot water-to-air heating system. The performance of controllers in regulating the discharge air temperature and flow rate is verified using the experimental system.

While simple optimal and SISO robust HVAC controllers have been designed and their performance experimentally verified, this project does so using MIMO robust controllers. The design and testing of these controllers provides valuable insight into potential improvements in performance, as well as constraints, associated with applying this control methodology to HVAC systems. Test results demonstrated that performance gains (reductions in discharge air temperature settle time) in excess of 300% may be achieved. Furthermore, it may be possible for such performance gains to be achieved without significant impact to current HVAC system architecture (interconnection).

Index Terms—HVAC, MIMO, robust, H_∞ , control, experimental verification, discharge air system, MATLAB, Simulink, Realtime Workshop, Windows Target.

I. INTRODUCTION

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Fig. 1. The Experimental HVAC System

IN commercial heating, ventilating, and air conditioning (HVAC) systems, a *central air supply* provides air at a controlled temperature and flow rate for use in heating (or cooling) a space. A heating coil is used in the central air supply for heating the discharged air. The temperature of the discharged air is controlled by regulating the rate at which hot water flows through the heating coil. The flow rate of the discharged air is regulated to maintain a predetermined static air pressure within the temperature controlled space. Typically, the space within a building is divided into smaller *zones*, allowing the temperature within each zone to be maintained independently of the others. Each zone contains a reheat coil which is used to moderate the final temperature of the air discharged into the zone.

A characteristic of today's HVAC systems is the use of a centralized hot water supplies in servicing multiple central air supplies. Such an HVAC interconnection (architecture) restricts the controller design and potentially impacts the performance of the resulting system. A *full* MIMO controller would require control of the temperature and flow rates of both the air and water flowing through the heating coil. Consequently, independent air and water supplies would be required for each coil. Such a system would represent a major shift from current HVAC design paradigms.

A problem with current HVAC controller design is that individual HVAC subsystems are treated as uncoupled (or

Fig. 2. Diagram of the Experimental System and Interface Signals

TABLE I
KEY TO MNEMONICS

C_{dr}	Command damper return	T_{ai}	Temp. of air input
C_{de}	Command damper external	T_{ao}	Temp. of air output
C_{wh}	Command water heater	T_{ws}	Temp. of water supply
C_{vp}	Command valve position	T_{wi}	Temp. of water into coil
C_{bs}	Command blower speed	T_{wo}	Temp. of water out of coil
		T_{ae}	Temp. of air external
A_{dr}	Actual damper return	T_{ar}	Temp. of air return
A_{de}	Actual damper external	F_w	Flow rate of water (coil)
A_{vp}	Actual valve position	F_{ws}	Flow rate of water (system)
P_{wh}	Power (input) water heater	F_a	Flow rate of air

loosely coupled) systems, controlled using distributed SISO proportional-plus-integral (PI) controllers, when in fact an HVAC system is a truly multivariable system. This article approaches HVAC controller design from a multivariable point of view and is one of the first to implement multivariable controllers on a physical HVAC system.

Between today's HVAC systems employing distributed SISO PI based controllers and a system using a *full* MIMO robust controller, a wide assortment of configurations are possible. To gain insight into potential performance improvements, as well as the constraints associated with this breadth of controllers, several diverse designs are implemented and tested, both in simulation and on the experimental HVAC system.

II. EXPERIMENTAL HVAC SYSTEM

The experimental HVAC system shown in Fig. 1 was constructed for verifying the performance of the advanced HVAC controller designs. This system consisted of external and return air dampers, a variable speed blower and a heating coil, which are similar to commercial hot water-to-air heating systems. A diagram representing this system is shown in Fig. 2 with the mnemonics defined in TABLE I.

The temperature of the discharged air was a function of the temperature and flow rate of both the air and water flowing through the coil. The flow rate of the air was primarily a function of the speed at which the blower is operating, but was affected slightly by the position of the return and external dampers. The dampers were electronically "ganged" together, allowing the return and external (outside) air mix to be varied, in regulating the temperature of the air flowing into the coil. A three-way mixing valve allows the flow rate of the water through the coil to be varied.

Fig. 3. General Feedback Interconnection for μ Synthesis

Controllers for the experimental system were implemented using *MATLAB*® and *Simulink*® with the *Real-Time Workshop*® (RTW) and *Windows Target*® toolboxes on a *Windows98*® based PC fitted with standard data acquisition cards. The design, construction, commissioning and modeling of this system is covered in previous works [1], [2].

III. SOME KEY POINTS REGARDING ROBUST CONTROL

While it is not possible to provide a complete tutorial on robust control theory in the space allowed, the following introduces a few of the key concepts relative to our approach. Inevitably, the models used in controller design only approximate the physical systems they intend to represent. *Robust control* theory addresses the affects that discrepancies between the model and the physical system (model uncertainty) may have on the design and performance of (linear) feedback systems. These uncertainties may arise from model approximation, neglected or unmodelled dynamics, unknown parameters, or even sensor and/or actuator imperfections. The techniques introduced here are based upon the structured singular value (μ) [10], which is frequently used in dealing with model uncertainty. Robust control provides a unified design approach under which the concepts of gain margin, phase margin, tracking, disturbance rejection and noise rejection are generalized into a single framework. Typically, the uncertainties considered in robust control theory are bounded using norms. The $\mathcal{H}_\infty$ and $\mathcal{H}_2$ norms are frequently applied in the robust controller design process [12], as they may be used to bound, respectively, the magnitudes or the energy content of signals. Herein, only the $\mathcal{H}_\infty$ robust control design methodology is considered.

To facilitate the task of controller synthesis, the general configuration shown in Fig. 3 is adopted. The block labelled P is the generalized plant model, which contains the nominal open-loop plant model along with the performance and uncertainty weighting functions. The block labelled Δ is block-diagonal complex uncertainty set (i.e. $\Delta \in \mathbf{\Delta}$, where $\mathbf{\Delta} = \{\Delta : \Delta = \text{block diag}(\Delta_1, \dots, \Delta_n)\}$). The individual Δ_i 's, are defined to be any stable transfer function with $\|\Delta_i\|_\infty \leq 1$, for $i = 1, \dots, n$, where $\|\cdot\|$ denotes the $\mathcal{H}_\infty$ norm. This implies that the Nyquist plot for each $\Delta_i(j\omega)$ is entirely contained within a circle having a radius of one, centered at the origin. The block labelled K represents the controller. The generalized plant model (P) has three input vectors: perturbations (u_Δ), disturbances (w) and control (u) and three output vectors: perturbations y_Δ , errors (z) and measurements (y).

The application of μ in robust control theory relies heavily upon a class of linear feedback loops called *linear fractional*

transformations (LFTs). If P is a 2×2 block-partitioned matrix and Δ is a matrix such that $\det(I - P_{11}\Delta) \neq 0$, then the upper LFT is defined $F_U(P, \Delta) = P_{22} + P_{21}\Delta(I - P_{11}\Delta)^{-1}P_{12}$. This LFT defines the perturbed open-loop transfer function from w to z . Similarly, if K is a matrix such that $\det(I - P_{22}K) \neq 0$, then the lower LFT is defined $F_L(P, K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}$. This LFT defines the nominal closed-loop transfer function from w to z . LFTs provide a general framework for defining a system interconnect. For more information regarding LFTs see [17], [3].

The set of perturbed plants (Π) to be controlled is defined by the LFT in eqn (1), where $\bar{\sigma}(\Delta)$ refers to the maximum singular value of Δ . For more information regarding the general configuration see [12].

$$\Pi = F_U(P, \Delta) : \Delta \in \mathbf{\Delta}, \sup_{\omega} \bar{\sigma}[\Delta(j\omega)] < 1 \quad (1)$$

The robust controller design objective is to find a controller (K) that, under all normalized uncertainties ($\|\Delta\|_{\infty} < 1$), stabilizes the perturbed closed-loop system and satisfies eqn (2).

$$\|F_L(\underbrace{F_U(P, \Delta)}_{\Pi}, K)\|_{\infty} \leq 1 \quad (2)$$

The standard $\mathcal{H}_{\infty}$ optimal control problem is to find all stabilizing controllers K that minimize (3).

$$\|F_L(P, K)\|_{\infty} = \sup_{\omega} \bar{\sigma}(F_L(P, K)(j\omega)) \quad (3)$$

There currently is no method that directly synthesizes an $\mathcal{H}_{\infty}$ robust controller; however, for complex perturbations, a method known as *DK-iteration* provides a good approximation. *DK-iteration* is a mix of $\mathcal{H}_{\infty}$ -synthesis and μ -analysis. An upper bound on μ , denoted as μ_{UB} , is given in eqn (4), where $\mathcal{D}$ is the set of scaling matrices having the property $D\hat{\Delta} = \hat{\Delta}D$ for every $D \in \mathcal{D}$, $\hat{\Delta} \in \mathbf{\Delta}$.

$$\mu_{UB} : \mu_{\hat{\Delta}}(F_L(P, K)) \leq \inf_{D \in \mathcal{D}} \bar{\sigma}(DF_L(P, K)D^{-1}) \quad (4)$$

Here, we can consider a robust controller that minimizes the peak value of μ_{UB} over frequency. This is expressed more precisely by eqn (5).

$$\min_{\substack{K \\ \text{stabilizing}}} \left(\min_{\substack{D \in \mathcal{D} \\ \text{stable, min-phase}}} \|DF_L(P, K)D^{-1}\|_{\infty} \right) \quad (5)$$

The *DK-iteration* process, which attempts to approximately solve eqn (5), is to alternate between minimizing $\|DF_L(P, K)D^{-1}\|_{\infty}$ with respect to either K or D , while holding the other constant. In practice, this method often converges to a μ -optimal controller. While μ -synthesis is not the only way to synthesize a robust controller, it is the method chosen for this application. In the work presented herein, the task of controller design, including *DK-iteration*, was carried out using the MATLAB *μ -Analysis and Synthesis Tool Box*. For more information regarding robust control theory see [10], [14], [12], [17], [18].

IV. OVERVIEW OF HVAC CONTROL

Early investigations regarding HVAC control focused on distributed SISO PI controllers. It has long been recognized that the necessarily low gains and tedious and (sometimes) inaccurate tuning of PI-based HVAC controllers contributed to poor performance [11], [9], [6]. Also, SISO controllers are unable to take advantage of the multivariable interactions in an HVAC system, which makes MIMO controllers more attractive.

Optimal control has been proposed as a means for MIMO based controller design (e.g., [15], [7], [16]); however, an optimal controller's sensitivity to discrepancies between the model used in designing the controller and that of the physical plant (model uncertainty) poses a major problem in HVAC systems, where accurate models are not readily available. Furthermore, the characteristics of an HVAC plant change (deteriorate) over time. Nonetheless, several optimal HVAC controller designs have been simulated and one has been tested on a simple system [7].

The application of *robust control* theory in controlling HVAC systems has only recently been considered. It has been proposed as a means to account for and compensate for both the uncertainty associated with the model used in controller design and the nonlinear nature of HVAC systems [4]. SISO robust controllers have been implemented and their performance verified in simulation [13] and on an experimental HVAC system [8]. Here, MIMO robust controllers for a simple HVAC application are implemented and tested.

V. MIMO ROBUST HVAC CONTROLLER DESIGN

To simplify the formulation, controller designs for the experimental HVAC system were restricted to *discharge air temperature and airflow rate control* (i.e. command T_{ao} and F_a to track reference inputs). The resulting *discharge air system* (DAS) is similar to the central air supply in a commercial HVAC system.

The four key control variables in a DAS are the input air and water temperatures (T_{ai} and T_{wi}) and flow rates (F_a and F_w) supplied to the heating coil. All four of these variables are controlled in a *full* MIMO DAS. However, in current HVAC systems, a common water supply typically provides hot water to multiple heating coils (i.e. T_{wi} is held constant). Thus, the water supply temperature cannot be varied at each DAS.

The design of an H_{∞} robust controller involves selecting frequency dependent weighting functions (transfer functions) that define the model uncertainty and the H_{∞} optimization criteria. To facilitate the design of these weighting functions, the experimental system model was arranged in a *canonical form* for controller synthesis, which is shown in Fig. 4.

This canonical form isolates the exogenous inputs (w_1 and w_2) and exogenous outputs (z_1 and z_2). The exogenous inputs are the weighted external disturbances and commanded inputs (w_1) and the weighted sensor noise measurements (w_2), an inevitable component of measurement signals. The exogenous outputs are the weighted error signals to be minimized (z_1) and the weighted controller outputs (z_2). Equation (6) shows

Fig. 4. System Model Configuration for Controller Synthesis

the signals (used in the design of controller K_{R2}) associated with the vectors w_1, w_2, z_1, z_2, u and y .

$$\begin{aligned}
 w_1 &= \begin{bmatrix} T_{ae} \\ T_{ar} \\ \Delta F_a \\ rF_a \\ rT_{ao} \\ rT_{ws} \\ rT_{ai} \end{bmatrix} & z_1 &= \begin{bmatrix} errorF_a \\ errorT_{ai} \\ errorT_{ws} \\ errorT_{ao} \end{bmatrix} & u &= \begin{bmatrix} C_{vp} \\ C_{bs} \\ C_{dr} \\ C_{wh} \end{bmatrix} \\
 w_2 &= \begin{bmatrix} noiseF_w \\ noiseF_a \\ noiseT_{ai} \\ noiseT_{ws} \\ noiseT_{wi} \\ noiseT_{wo} \\ noiseT_{ao} \\ noiseT_{ae} \\ noiseT_{ar} \end{bmatrix} & z_2 &= \begin{bmatrix} C_{vp} \\ C_{bs} \\ C_{dr} \\ C_{wh} \end{bmatrix} & y &= \begin{bmatrix} F_w \\ errorF_a \\ errorT_{ai} \\ errorT_{ws} \\ T_{wi} \\ T_{wo} \\ errorT_{ao} \end{bmatrix}
 \end{aligned} \quad (7)$$

Note: vectors w_1, w_2, z_1 and z_2 are weighted (optimization criteria), vectors u and y are not weighted (physical signals).

In relation to Fig. 3, the exogenous inputs and outputs are the (stacked) vectors

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \quad \text{and} \quad z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \quad (8)$$

respectively, and the generalized plant (P) contains the nominal plant, weighting matrices, and Pade delay. The goal of μ -synthesis is to find a controller (K) such that

$$\left\| \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \right\|_{\infty} < 1 \quad \Rightarrow \quad \left\| \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \right\|_{\infty} < 1, \quad (9)$$

for all (stable) $\Delta \in \mathbf{\Delta}$, $\|\Delta\|_{\infty} < 1$, assuming one exists. Note that the inputs and outputs are normalized (bounded by one in magnitude). This is the standard arrangement for μ -synthesis. Note that this normalization is not restrictive, since the physical signals may be scaled through the use of weighting functions. For example, if the design requires that an output signal be less than 0.2 in magnitude (in a given frequency range), then the weight associated with that signal would have a magnitude of 5 (in the given frequency range). This is the crux of the design process, which will be discussed in this section and the next.

As the structure of the controller is embedded in the nominal plant model, many controller architectures may be realized using the generalized model shown in Fig. 4. The signals

comprising the vectors w_1, w_2, z_1, z_2, u and y depend upon this structure. The weighting matrices in Fig. 4 are allowed to have a full structure; however, in the context of this paper, they will be assumed to be diagonal matrices. This is done to so that each signal has only one weight describing its uncertainty or optimization criteria, which provides a more intuitive design process. As the design is performed in continuous-time and the controller implemented in discrete-time, *Pade* delays are used in the feedback loop to include the effects of sampling. All of the MIMO robust controllers presented in this chapter conform to the system model of Fig. 4.

A. Weight Selection

The selection of weights is a major part of the controller design process. For the experimental system shown in Fig. 4, there were two types of weights that needed to be specified, namely model uncertainty weights (W_i and W_o) and optimization criteria weights (W_{dist} (disturbance), W_{perf} (performance), W_{noise} (sensor noise), and $W_{control}$ (control)). The model uncertainty weights describe how accurate the model is at various frequencies. For the MIMO robust controller designs presented herein, additive uncertainty weights were used. The optimization criteria weights were used to limit the size of the exogenous signals. When choosing these weights, there was a tradeoff between performance (i.e. good tracking and large control signals) and robustness (i.e. poor tracking and small control signals). For this application, performance was required at lower frequencies in exchange for robustness at higher frequencies. This meant that most of the weighting functions took the form of either lowpass filters (LPFs) or highpass filters (HPFs).

All of the lowpass and highpass filters used were first order filters. The LPFs were designed using the transfer function

$$W_L(s) = \frac{g\omega_c}{s + \omega_c}, \quad (10)$$

where g was the approximate passband gain and ω_c was the cutoff frequency in radians per second (rad/sec). The HPFs were designed using the transfer function

$$W_H(s) = \frac{g_{\infty}(s + \omega_L)}{s + \frac{g_{\infty}}{g_o}\omega_L}, \quad (11)$$

where g_o was the approximate stopband gain that extends from DC (zero rad/sec) to ω_L (rad/sec), and g_{∞} was the approximate passband gain for frequencies above $\omega_H = \frac{g_{\infty}}{g_o}\omega_L$. These weights were designed by specifying the stopband cutoff frequency (ω_L). Note that the size of the transition band (ω_L to ω_H) is directly proportional to the ratio $\frac{g_{\infty}}{g_o}$.

The selection of weighting functions was not purely an analytic procedure, but rather an iterative process of initial estimation, testing, and revision. The initial weights are chosen based upon knowledge of the system or signals (e.g., noise amplitude and frequency, actuation limits, measured responses, etc.), and a controller was synthesized. Based on the performance in simulation, the weights were tweaked to provide a good response. Once a controller was working well in simulation, it was implemented on the experimental

system. Based on these results, the controller weights were again tweaked, simulated in software, and implemented on the experimental system. To this extend, the weights were regarded as *tuning parameters*.

As mentioned earlier, only diagonal weight matrices were used. The diagonal elements of these weight matrices were the transfer functions (as in eqns (10) and (11)) that described the frequency based weight for a given signal. In the μ -synthesis software used, these weighting matrices were never explicitly defined. Instead, they were implicitly defined by the system interconnect. In the next two subsections, the design methodology for the model uncertainty weights and the optimization criteria weights is addressed. An exemplar MIMO robust controller design is given in Section VIII.

B. Model Uncertainty Weights

The HVAC system contained five subsystems, namely the *value*, the *blower*, the *mixing box*, the *boiler*, and the *heating coil*. A careful inspection of Fig. 2 will reveal these subsystems. Additive uncertainty was incorporated around each of these five subsystems.

Model uncertainty weights are usually either HPF or constant weights (i.e. allpass filters). HPF are generally used when the model is assumed to be very accurate at low frequencies (i.e. a small magnitude means less uncertainty) and may not be very accurate at high frequencies (i.e. a larger magnitude means more uncertainty). This generally gives a more accurate uncertainty description, but will increase the order of the synthesized controller [12]. When the uncertainty description of a particular input or output does not appear to have a great affect on the final μ plot (or for ease of design), constant weights may be used. This method will generally assume a larger model uncertainty than is necessary at lower frequencies, but will result in a synthesized controller of lower order.

C. Optimization Criteria Weights

The HVAC controller designs presented herein were designed to have performance at “low” frequencies and robustness at “high” frequencies. Due to the slow dynamics of the HVAC system, “low” frequencies were in the DC to 10 mHz range and “high” frequencies were 5 Hz and higher. In between these frequency ranges was the transition region where the weights changed from performance specifications to robustness specifications. Choosing how the weights transition in this region will have an affect on how well the controller performs. For the experimental system used, the focus of the transition band was in the 0.5 Hz to 2 Hz range. The optimization criteria weights were used to manipulate the H_∞ controller synthesis under the constraint given in eqn (9). In some cases, these weights were used to limit the size of the physical (performance) signals (i.e. control and error signals), and in other cases, they were used to represent the magnitude of the physical (external) signals (i.e. disturbance and noise signals). The design of these weights will be discussed in more detail in the next four sections.

1) *Disturbance Weights*: In the HVAC setup considered herein, there were three types of disturbances, namely external temperature measurements (T_{ae} and T_{at}), variations in the flow rate of air (ΔF_a), and reference inputs. Since disturbances occurred at “lower” frequencies, LPFs were used to represent them. Intuitively, these LPFs provide performance (i.e. the controller responds to disturbances) at lower frequencies and robustness (i.e. the magnitude of the scaled disturbance is so small the H_∞ optimization ignores the signal) at higher frequencies.

2) *Performance Weights*: The performance weights and the disturbance weights were closely related, since they effectively scaled the same signals. The performance weights scaled the error signals of the tracked reference inputs (and the disturbance weights scaled the reference inputs). Since performance was sought at “lower” frequencies (i.e. good performance at DC means good tracking in steady state), LPFs were employed. Even more so, the LPFs were stable approximations to integrators (i.e. first-order strictly proper transfer functions with a pole in the 10 μ Hz to 100 μ Hz range). This was done since the HVAC system was designed to track step changes. Over any given frequency range, the amount of tracking obtained (for a given reference input) is determined from the product of a disturbance weight and the associated performance weight.

3) *Sensor Noise Weights*: The sensors were used to provide a signal representation of the measured system variables, which inevitably contain noise. These sensors were more accurate at lower frequencies and not as accurate at high frequencies. Therefore, these weights were chosen to be HPFs. In order for the H_∞ work properly, these weights must be semi-proper (i.e. they must have a non-zero gain at high frequencies).

4) *Control Weights*: Since the actuation rates and ranges of all control devices are finite, the magnitude and frequency response of their corresponding control signals must be limited. For these weights, a larger magnitude (in a given frequency range) will force the H_∞ optimization routine to limit the amount of control allowed (in that frequency range). Therefore, a smaller magnitude is needed at lower frequencies where performance is sought (i.e. the controller must be able to exert control at lower frequencies) and a larger magnitude is needed at higher frequencies where robustness is sought. These controller objectives were satisfied using HPFs.

VI. OVERVIEW OF HVAC CONTROLLER DESIGNS

The four controllers considered herein were intended for controlling HVAC systems having diverse architectures. They range from a PI based controller (K_{PI}), for controlling systems with a shared water supply, to a *full* MIMO robust controller (K_{R3}) requiring separate water supplies for each DAS. The system architecture and controller features considered are summarized in TABLE II. The four controllers are specified in the first (left) column. Each “major” row contains data pertinent to each of the four controller designs, while the five right-most columns contain data relative to the key system variables: water supply temperature (T_{ws}), flow rate of

TABLE II

SUMMARY OF HVAC SYSTEM AND CONTROLLER ARCHITECTURES

Controller	Water Supply		Air Supply		Temp. Air Out (T_{ao})
	T_{ws} (T_{wi})	F_w	T_{ai}	F_a	
K_{PI} 3 : SISO	SISO (PI)	$\Rightarrow T_{ao}$	SISO (PI)	Open-loop	SISO (PI)
	Tracks rT_{ws}	Varied to Control T_{ao}	Tracks rT_{ai}	Manually Controlled	Tracks rT_{ao}
K_{R1} MIMO (3 × 6) plus one SISO	SISO (PI)	MIMO Controlled			
	Tracks rT_{ws}	Free Variable	Free Variable	Tracks rF_a (at low freq.)	Tracks rT_{ao}
K_{R2} MIMO (4 × 7)	MIMO Controlled				
	Tracks rT_{ws}	Free Variable	Tracks rT_{ai} (at low freq.)	Tracks rF_a (at low freq.)	Tracks rT_{ao}
K_{R3} MIMO (4 × 7)	MIMO Controlled				
	Free Variable	Free Variable	Tracks rT_{ai}	Tracks rF_a (at low freq.)	Tracks rT_{ao}

Key: * dynamic participation only (tracks reference signal in steady-state)

water through the coil (F_w), temperature of air into the coil (T_{ai}), air flow rate (F_a) and temperature of air out (T_{ao}). The most important variables being regulated, air flow rate (F_a) and output air temperature (T_{ao}), appear in the two right-most columns. For each controller (major-row, including sub-rows), each column contains data pertaining to one of the five key variables. The upper sub-row for each controller, identifies the controller regulating each of the five variables. This is significant for the first two controllers, K_{PI} , and K_{R1} , where the “overall” DAS controller consists of multiple controllers. The lower sub-row for each controller contains descriptive information regarding the control variable (e.g., if the variable tracks a reference signal, and if so, only at low frequencies, or if it is a “free” control variable). For the output air temperature (right-most) column, the lower sub-row also indicates what variables are used in controlling the output air temperature ($f(\cdot)$).

As indicated in TABLE II, the reference controller K_{PI} consisted of three SISO (PI) controllers tracking three reference signals. They were used in regulating the water supply temperature (T_{ws}), input air temperature (T_{ai}) and output air temperature (T_{ao}). While the air flow rate (F_a) could easily have been controlled in a similar manner, it was adjusted manually (in open loop) by varying the blower speed (C_{bs}), as air flow rate (F_a) was virtually a direct function of the blower speed. The output air temperature (T_{ao}) was controlled by a PI controller which varied the valve position (C_{vp}) to regulate the flow rate of water (F_w) through the coil.

The structure of controller K_{R1} , provided a “minimal” MIMO robust controller compatible with a system in which the water supply may be shared between multiple DASs. As shown in TABLE II, the overall controller consisted of a 3 × 6 MIMO controller and a SISO PI controller that was used to regulate the water supply temperature. The MIMO controller tracked two reference signals, namely rF_a (at low frequencies) and rT_{ao} , and had two free control variables, namely the temperature of the air flowing into the coil (T_{ai}) and the rate at which the water flows through the coil (F_w). Assuming a constant water supply temperature (F_{ws}), the output air temperature (T_{ao}) was a function of the water flow rate (F_w), the input air temperature (T_{ai}), and the air flow rate (F_a).

Controller K_{R2} was a “constrained” MIMO robust controller. This controller was similar to K_{R1} in the sense that the water flow rate through the coil (F_w) was a “free” control variable, and that the output air temperature (T_{ao}) was a function

of the water flow rate (F_w), the input air temperature (T_{ai}), and the air flow rate (F_a). Unlike K_{R1} , this controller regulated the input air temperature to track a reference input (rT_{ai}) and incorporated the control of the water supply temperature into the MIMO controller. Like K_{R1} , this controller held the shared water supply for all DASs at a constant temperature. This controller would be the most applicable to current HVAC setups where one MIMO controller would be used to regulate the airflow rate (F_a) and discharge air temperature (T_{ao}) in each zone while maintaining a constant shared water supply temperature.

The last controller K_{R3} , was a “full” MIMO robust controller and in a system having multiple DASs, would require an independent (separate) water supply for each DAS. In this controller, the (heater command output controlling the) water supply temperature was a “free variable” allowing the water supply temperature to vary. The input air temperature was controlled to track the reference input (rT_{ai}) during steady state, but was allowed to vary some during transitions. This controller was the only MIMO robust controller that used dynamic model uncertainty weights (i.e. HPFs were used instead of constant weighting functions). Only a minimal amount of improvement was achieved by using these weights.

In the next two sections, the design of the reference PI controller and the K_{R2} MIMO robust controllers will be discussed in detail. A similar design procedure was used for designing the other MIMO robust controllers, only the design of K_{R2} will be discussed in detail. For a detailed discussion of the K_{R1} and K_{R3} designs, see [1].

VII. REFERENCE PI CONTROLLER DESIGN

Contemporary HVAC controllers utilize multiple PI controllers to regulate individual SISO control loops. Since the blower fan already had built-in controller (variable frequency drive), the airflow rate (F_a) was controlled by commanded blower speed (C_{bs}). Therefore, only three (SISO) PI controllers, namely $K_{PI}^{T_{ws}}$, $K_{PI}^{T_{ai}}$, and $K_{PI}^{T_{ao}}$, were used to control the commanded water heater temperature (C_{wh}), commanded return (and hence external) air damper position (C_{dr} and hence C_{de}), and commanded valve position (C_{vp}), respectively. This is illustrated in Fig. 5.

Fig. 5. HVAC Controller Based Upon Three SISO PI Controllers

TABLE III
PROPORTIONAL AND INTEGRAL GAINS FOR PI CONTROLLERS

PI Controller	Proportional Gain(K_p)	Integral Gain(K_i)
(K_{PI}^{Tai})	0.20	0.010
(K_{PI}^{Tao})	0.24	0.025
(K_{PI}^{Tws})	1.80	0.015

Though providing tolerable performance, the PI control faces several challenges. First, the subsystems that they control have gains that vary nonlinearly. Thus, the PI controller must be tuned at the highest gain state to maintain stability over the entire range. This provides a slow response in low gain states. This makes tuning the PI controller time consuming and tedious, with optimal performance rarely attained. Nonetheless, considerable effort was put into the design of these controllers to get the best possible response on the physical system. Specifically, these gains were tuned for the experimental HVAC system using the method outlined in [5, p. 285]. The final controller gains are given in TABLE III.

Windup is another problem common in HVAC systems, which occurs when the controller states become inconsistent with the control signals (e.g., a linear control drives an actuator into saturation). Following such an occurrence, the control loop may experience severe transients. The PI controller design presented herein employed an *antiwindup* technique to avoid this problem.

VIII. MIMO ROBUST CONTROLLER K_{R2} WITH REFERENCED WATER SUPPLY TEMPERATURE CONTROL DESIGN

The first step in designing the K_{R2} controller was to determine the system and controller architectures. Controller K_{R2} controlled the water supply temperature (T_{ws}), input air temperature (T_{ai}), air flow rate (F_a) and the discharge air temperature (T_{ao}). A system interconnect incorporating this controller is shown in Fig. 6.

The K_{R2} controller synthesis task required a more detailed description than the canonical synthesis model shown in Fig. 4. This was accomplished by creating a textual description (defined using *sysic*, a component of the MATLAB *μ -Analysis and Synthesis Toolbox*) of the model (given in [1]) that defined the overall HVAC system interconnect and associated weighting functions. The crux of the design is to select the frequency dependent weights, which are discussed in detail in the next two subsections.

A. Model Uncertainty Weight Selection

Additive uncertainty was incorporated around each of the five subsystems. For each subsystem, the number of additive uncertainty weights was determined by the number of physical inputs and outputs. The number of inputs and outputs for each subsystems determined the size of individual blocks in $\Delta = \text{diag}(\Delta_V, \Delta_B, \Delta_M, \Delta_H, \Delta_C)$. Note that each subsystem had its own uncertainty block.

Fig. 6. Experimental System Using MIMO Robust Controller K_{R2}

The model uncertainty for each subsystem was split into uncertainty on the inputs (W_i) and uncertainty on the outputs (W_o). While it is possible to lump all of the uncertainty into either the input or the output, splitting up uncertainty description will give a more balanced H_∞ design and a more intuitive weight selection process.

In some applications, the μ -synthesized controller is very sensitive to model uncertainty weights; however, for the K_{R2} controller design, it was only important that *some* uncertainty was included for each input and output signal. Therefore, constant model uncertainty weights were used. Since the overall uncertainty for a subsystem is (effectively) determined by the product of the input and output weights, each of the five subsystems had the same effective uncertainty description, namely the inputs had either a weight of 0.001 or 0.005 and the outputs had either a weight of 0.005 or 0.001, respectively. The weights for each subsystem is summarized in TABLE IV.

As mentioned earlier, the matrices W_i and W_o were never explicitly defined. Instead, the weights were embedded into the system interconnect (using *sysic*) and only the block structure, which was determined from the physical interconnect, was given to the μ -synthesis software. The particular block diagonal structure for K_{R2} is given in eqn (12).

$$\Delta_{\text{BlkStr}} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 2 & 4 \end{bmatrix} : \begin{array}{l} \text{blk str for } \Delta_V \text{ Value} \\ \text{blk str for } \Delta_B \text{ Blower} \\ \text{blk str for } \Delta_M \text{ MixingBox} \\ \text{blk str for } \Delta_H \text{ Heater} \\ \text{blk str for } \Delta_C \text{ Coil} \end{array} \quad (12)$$

This structure should be evident by inspecting TABLE IV. From this, W_i is a matrix whose diagonal elements are the input weights in TABLE IV, and W_o is a matrix whose diagonal elements are the output weights in TABLE IV.

TABLE IV
MODEL UNCERTAINTY WEIGHTS USED IN THE DESIGN OF K_{R2}

<i>Valve Uncertainty</i>			
$w_{V_{in}}(C_{vp})$	= 0.001	$w_{V_{out}}(F_w)$	= 0.005
		$w_{V_{out}}(F_{ws})$	= 0.005
<i>Blower Uncertainty</i>			
$w_{B_{in}}(C_{bs})$	= 0.001	$w_{B_{out}}(F_a)$	= 0.005
$w_{B_{in}}(C_{dr})$	= 0.001		
<i>Mixing Box Uncertainty</i>			
$w_{M_{in}}(C_{dr})$	= 0.005		
$w_{M_{in}}(T_{ar})$	= 0.005	$w_{M_{out}}(T_{ai})$	= 0.001
$w_{M_{in}}(T_{ae})$	= 0.005		
<i>Water Heater (Boiler) Uncertainty</i>			
$w_{H_{in}}(C_{wh})$	= 0.005		
$w_{H_{in}}(T_{wo})$	= 0.005	$w_{H_{out}}(T_{ws})$	= 0.001
$w_{H_{in}}(F_w)$	= 0.005		
$w_{H_{in}}(F_{ws})$	= 0.005		
<i>Heating Coil Uncertainty</i>			
$w_{C_{in}}(F_a)$	= 0.005		
$w_{C_{in}}(F_w)$	= 0.005	$w_{C_{out}}(T_{wo})$	= 0.001
$w_{C_{in}}(T_{ai})$	= 0.005	$w_{C_{out}}(T_{ao})$	= 0.001
$w_{C_{in}}(T_{wi})$	= 0.005		

B. Optimization Criteria Weight Selection

The optimization criteria weights were used to scale the normalized inputs and outputs so that they accurately represent the relative magnitudes that are seen by the HVAC system. When a weight was too small, the H_∞ controller synthesis software assumed that the signal would not have a large enough magnitude to affect the overall stability. This often resulted in a controller that is over-reactive (e.g. if too much control authority is given to a particular control signal). Similarly, when a weight was too large, the H_∞ controller synthesis software put too much effort into minimizing the gain of a signal that the physical system would never see. This usually lead to a lack of performance. Ultimately, the magnitudes and frequency ranges of the weights had to be tuned until the fastest response (while maintaining disturbance rejection) was obtained on the experimental system. While it is not possible to discuss every minor change made to the weights, the justification for the final weights is given in the next four sections.

1) *Disturbance Weights:* For the K_{R2} controller, the disturbances (w_1) consisted of air temperatures from the surrounding environment (T_{ae} and T_{ar}), variations in the flow rate of air (ΔF_a) due to wind gusts, and commanded inputs (rT_{ao} , rF_a , rT_{ai} , and rT_{ws}). The air temperatures from the environment generally did not change vary rapidly, so the disturbance weights associated with them had a bandwidth of 3 mHz. Since these disturbances did not affect the output as much as the reference inputs, a gain of 0.5 was used. The wind gusts were assumed to have an even smaller affect on the outputs, so its magnitude was 0.005 in the passband. This weight covered wind gusts in the DC to 0.32 Hz (2 rad/sec) range, which was enough bandwidth to “cover” the wind gusts seen in experimental testing. The reference input weights all had a cutoff frequency of 0.16 Hz (1 rad/sec), which was reasonable

TABLE V
DISTURBANCE (& REFERENCE) WEIGHTS USED IN THE DESIGN OF K_{R2}

$w_{Dist}(T_{ae})$	= $\frac{0.01}{s+0.02}$	$w_{Dist}(T_{ar})$	= $\frac{0.01}{s+0.02}$
$w_{Dist}(\Delta F_a)$	= $\frac{0.01}{s+2}$	$w_{Dist}(rF_a)$	= $\frac{0.8}{s+1}$
$w_{Dist}(rT_{ao})$	= $\frac{5}{s+1}$	$w_{Dist}(rT_{ws})$	= $\frac{1}{s+1}$
$w_{Dist}(rT_{ai})$	= $\frac{1}{s+1}$		

TABLE VI
PERFORMANCE WEIGHTS USED IN THE DESIGN OF K_{R2}

$w_{Perf}(F_a)$	= $\frac{0.1}{s+0.0002}$	$w_{Perf}(T_{ao})$	= $\frac{0.005}{s+0.00025}$
$w_{Perf}(T_{ai})$	= $\frac{0.001}{s+5 \times 10^{-5}}$	$w_{Perf}(T_{ws})$	= $\frac{0.02}{s+0.004}$

since the highest frequency allowed for controlling the value and the dampers was about 0.1 Hz (see the section on control weight selection). Since the reference signals were the largest disturbance signals, they had the largest passband magnitudes. The transfer functions used to describe these weights are given in TABLE V.

2) *Performance Weights:* The performance weights (w_{Perf}) were chosen to specify the performance objectives that the controller was to attain. In particular, good steady state tracking was desired, so these weights had large values at DC and then rolled off above 0.1 mHz. The transfer functions describing these weights are given in TABLE VI. The approximate error allowed in steady state for each commanded input was determined by taking the reciprocal of the product of the reference disturbance weight and performance weight at DC (e.g. $w_{Dist}(rT_{ao}) = 5$ and $w_{Perf}(T_{ao}) = 20$ at DC; therefore, the temperature air out tracking error (eT_{ao}) was less than 0.01 in steady state). The resulting tracking errors allowed in steady stage for the commanded inputs were $eF_a < 0.0025$, $eT_{ws} < 0.2$, $eT_{ai} < 0.05$, and $eT_{ao} < 0.01$. NB: These error bounds are in the units of the physical signals (i.e. flow rate of air is in $\frac{m^3}{s}$ and temperature is in $^\circ C$).

3) *Sensor Noise Weights:* Sensor noise is typically a high frequency phenomenon (i.e. sensors are assumed to be most accurate at frequencies around DC). Therefore, these weights were chosen to be HPFs. When possible, these weights are chosen based on the sensor product description. While the magnitude of these weights are generally small and the weights generally do not have a large impact on the synthesized controller, they play an important role, namely they guarantee that the H_∞ optimization is well posed. This is due to the fact that that the weights have non-zero magnitude at high frequencies. For more information regarding this, see [12]. The transfer functions describing the sensor noise weights (w_{SN}) are given in TABLE VII.

4) *Control Weights:* The control weights (w_{Cont}) had the most impact on the synthesized controller. The flow control value and the dampers were pneumatically controlled, which resulted in them responding the slowest to step changes.

TABLE VII
SENSOR NOISE WEIGHTS USED IN THE DESIGN OF K_{R2}

$w_{SN}(F_w) = \frac{10s+1 \times 10^{-6}}{1 \times 10^6 s+1}$	$w_{SN}(F_a) = \frac{0.1s+0.001}{0.1429s+1}$
$w_{SN}(T_{ai}) = \frac{0.1s+0.001}{0.1s+1}$	$w_{SN}(T_{ws}) = \frac{0.05s+0.01}{0.05s+1}$
$w_{SN}(T_{wi}) = \frac{0.05s+0.01}{0.05s+1}$	$w_{SN}(T_{wo}) = \frac{0.05s+0.01}{0.05s+1}$
$w_{SN}(T_{ao}) = \frac{0.1s+0.01}{0.1s+1}$	$w_{SN}(T_{ae}) = \frac{0.1s+0.01}{0.1s+1}$
$w_{SN}(T_{ar}) = \frac{0.1s+0.01}{0.1s+1}$	

TABLE VIII
CONTROL WEIGHTS USED IN THE DESIGN OF K_{R2}

$w_{Cont}(C_{vp}) = \frac{2s+0.015}{0.2s+1}$	$w_{Cont}(C_{bs}) = \frac{0.1592s+0.01}{0.01592s+1}$
$w_{Cont}(C_{wh}) = \frac{0.1592s+0.15}{0.001592s+1}$	$w_{Cont}(C_{dr}) = \frac{1.333s+0.02}{0.1333s+1}$

Therefore, their control inputs (C_{vp} and C_{dr}) were allowed to provide large control signals in the DC to 2 mHz range and were forced to be negligible above the 1 Hz range. These ranges were chosen to fit within the actuator limits for the valve and damper. The blower had a built-in controller that was able to change the flow rate of air (F_a) with the commanded blower speed (C_{bs}); however, high frequency changes in one variable disrupted the other outputs. Therefore, the commanded blower speed (C_{bs}) was allowed to have the largest control signals in the DC to 10 mHz range, and was forced to be negligible above 10 Hz. For the three weights discussed so far, the control weights were in the 0.01 to 0.02 magnitude range. For this system, weight values below 0.01 resulted in overactive controllers and weight values above 0.02 resulted in sluggish controllers (i.e. controllers that had a longer settling time). The fourth control variable was the commanded water temperature in the system (C_{wh}). The water heater required smaller control signals, so a larger weight (about ten times the size of the blower speed weight ($w_{Cont}(C_{bs})$)) was used. Both the blower speed weight ($w_{Cont}(C_{bs})$) and the water heater weight ($w_{Cont}(C_{wh})$) reached a magnitude of one around the 1 Hz range, which may be viewed as a loose upper bound to the fastest controllable dynamic in the HVAC system. The transfer functions describing the control weights are given in TABLE VIII.

C. K_{R2} Structured Uncertainty Description

The robust stability block diagonal structure (Δ) associated with the model uncertain was given in eqn (12). The structure for the nominal performance block was determined from the number of exogenous inputs and outputs defined in eqn (6) (i.e. the number of optimization criteria weights). The nominal performance block structure Δ_p had 8 inputs (i.e. the exogenous outputs (z) from the system were perturbation inputs) and 16 outputs (i.e. the exogenous inputs (w) into the system were perturbation outputs), which yielded the nominal performance block $\Delta_p \in \mathbb{C}^{16 \times 8}$. Together, these block structures were used to create the robust performance block $\hat{\Delta}$ in eqn (13).

$$\hat{\Delta}_{\text{BlkStr}} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 2 & 4 \\ 16 & 8 \end{bmatrix} \begin{array}{l} : \text{blk str for } \Delta_V \text{ Valve} \\ : \text{blk str for } \Delta_B \text{ Blower} \\ : \text{blk str for } \Delta_M \text{ MixingBox} \\ : \text{blk str for } \Delta_H \text{ Heater} \\ : \text{blk str for } \Delta_C \text{ Coil} \\ : \text{blk str for } \Delta_p \text{ (} z \text{ \& } w \text{ scaling)} \end{array} \quad (13)$$

NB: Only the system interconnect (with the individual weights defined) and the block structure in eqn (13) were required for μ -synthesis software. For more information on synthesizing μ -optimal robust controllers using the μ -Analysis and Synthesis toolbox, see [3] and [12].

IX. EXPERIMENTAL VERIFICATION OF CONTROLLERS

An experimental system for testing advanced HVAC controllers, was built using standard HVAC components. While only a portion of an overall HVAC system, this discharge air system (DAS) was representative of a typical hot water to air heating system. The construction and modeling of this system is the focus of another article [2].

A. Reference PI Controller Verification

A Simulink model in conjunction with MATLAB's *Real-time Workshop Toolbox* (RTW) was used to test the performance of the reference PI controller. This controller was first simulated in software and then compiled and downloaded onto the hardware, where it was used to control the experimental HVAC system in real-time. After the experimental system was running and had reached steady state under closed-loop control, it was subjected to a series of step changes in the output air temperature (rT_{ao}) and air flow rate (rF_a) reference inputs. The response of the experimental system to these inputs is shown in Fig. 7.

At 308 seconds, the output air temperature reference input (rT_{ao}) was increased (stepped) from 39.5°C to 44.5°C . The controller $K_{PI}^{T_{ao}}$ responded by increasing the valve position (C_{vp}) output (increasing the water flow rate (F_w)), which caused the temperature of the discharge air (T_{ao}) to rise. After 931 seconds elapsed, the reference level was reached. At 1,800 seconds, the air flow rate (rF_a using the C_{bs} output) was abruptly stepped down from $0.272 \frac{\text{m}^3}{\text{s}}$ (38.2% of the maximum rate) to $0.151 \frac{\text{m}^3}{\text{s}}$ (21.4%). In response to the rising temperature of the output air (T_{ao}), the controller $K_{PI}^{T_{ao}}$ reduced the valve position command (C_{vp}) (decreasing the water flow rate (F_w)). However, the output air temperature (T_{ao}) increased 1.65°C before the water flow (F_w) was reduced and the temperature returned to the reference level (rT_{ao}) approximately 1,400 seconds after initiation of the air flow rate change. At 3,400 seconds, the air flow rate (F_a) was returned to its original level, causing the output air temperature to drop 2°C before returning to the reference level 1,610 seconds after initiation of the air flow rate change. At 5,152 seconds, the output air temperature reference (rT_{ao}) returned to its original level (39.5°C). After roughly 931 seconds elapsed, the output air temperature (T_{ao}) decreased to this level.

These experimental results demonstrate that an HVAC system is *not* an uncoupled dynamical system but rather a multivariable system. Specifically, this is illustrated by the

Fig. 7. Controller K_{PI} Experimental Test Results

inverse response from increasing the airflow rate (F_a) which decreased the output air temperature (T_{ao}). To the controller $K_{PI}^{T_{ao}}$ is a disturbance, and it is well known that PI controllers are good at disturbance rejection. Therefore, well-tuned PI controllers are able to provide acceptable performance in commercial applications. However, the overall performance is limited by the PI controllers inability to make coordinated control actions. More performance may be achieved by using a multivariable controller that is able to make coordinated control actions. The next section illustrates an example MIMO

controller's ability to make coordinated control actions and the performance achieved as a result of coordinated control.

B. MIMO Robust Controller with Reference Water Supply Temperature Control Verification

The controller setup in Fig. 6 was converted to a Simulink diagram (with RTW) that was used to test the performance of controller K_{R2} . As with the PI controller, this controller was first simulated in software and then compiled and downloaded

onto the hardware, where it was used to control the experimental HVAC system in real-time. Unlike the PI controller, this HVAC system was brought to steady-state in open-loop, and the loop was closed using a (nearly) bumpless transfer technique. For details, see [1]. The closed-loop system (after the bumpless transfer had settled) was subjected to a series of step changes. The response of the experimental system to these inputs is shown in Fig. 8.

From comparing the results shown in Figs. 7 and 8, it is apparent that controller K_{R2} responded faster than the reference controller K_{PI} . As a detailed comparison of controller performance appears later, consider for the moment the experimental results for controller K_{R2} , shown in Fig. 8. At 306 seconds, the output air temperature (rT_{ao}) reference was increased from 39.5°C to 44.5°C . The controller responded by increasing both the valve position (C_{vp}) and water heater (C_{wh}) outputs, as well as (after first increasing) momentarily decreasing the blower speed output (C_{bs}) and briefly increasing the return air damper position output (C_{dr}). In response to these actions, the water flow rate (F_w) increased, the water supply temperature (T_{ws}) was maintained, the air flow rate (F_a) momentarily decreased, and the input air temperature (T_{ai}) increased briefly. The output air temperature (T_{ao}) reached the specified temperature 322 seconds after the step in the reference signal. Since the controller K_{R2} controlled the input air temperature to in steady-state, track a reference input (rT_{ai}), only momentary deviations were allowed. However, these momentary changes in input air temperature (T_{ai}) did aid in reducing the response time of the system.

At 800 seconds, the air flow rate reference (rF_a) was reduced from $0.29 \frac{\text{m}^3}{\text{s}}$ (40.7%) to $0.19 \frac{\text{m}^3}{\text{s}}$ (26.7%). The controller responded by rapidly reducing the blower speed output (C_{bs}) to 90%, then gradually to 100% of the new level. This quickly decreased the air flow rate to $0.2 \frac{\text{m}^3}{\text{s}}$ (28.1%), then gradually decreased to the commanded level in 381 seconds. Simultaneously, while reducing the air flow rate (F_a), the controller momentarily reduced the return air damper (C_{dr}) output and reduced both the water heater (C_{wh}) and the valve position (C_{vp}) outputs. These actions momentarily lowered the input air temperature, maintained the water supply temperature and reduced the water flow rate (F_w). At 1,590 seconds, the air flow rate reference was increased from $0.19 \frac{\text{m}^3}{\text{s}}$ (26.7%) to $0.39 \frac{\text{m}^3}{\text{s}}$ (54.7%). The control responded by rapidly increasing the air flow rate (C_{bs}) output (raising the air flow rate (F_a) to $0.37 \frac{\text{m}^3}{\text{s}}$ (51.9%)), momentarily increasing the return air damper (C_{dr}) output, and increasing both the water heater (C_{wh}) and valve positions (C_{vp}). The full air flow rate was reached in 307 seconds. At 2,678 seconds, the air flow rate reference (rF_a) was reduced to the original level ($0.29 \frac{\text{m}^3}{\text{s}}$). The control responded in a similar manner, as it did to the first step reduction in air flow rate, reaching the commanded level after 283 seconds.

At 3,380 seconds, the output air temperature reference (rT_{ao}) was reduced from 44.5°C to 39.5°C . The control responded in a manner similar, but opposite from that of the step increase in the output air temperature reference. After roughly 322 seconds, the reference output air temperature was

TABLE IX
COMPARISON OF CONTROLLER PERFORMANCE

Performance Measurement		Controller				units
		K_{PI}	K_{R1}	K_{R2}	K_{R3}	
T_{ao}	Rise Time	891	150	178	91	sec.
	Settle Time	931	275	322	218	sec.
	Overshoot	0.0	2.8	2.0	4.0	%
	Disturbance	96.0	96.4	98.3	96.3	%
	Rejection					%
F_a	Rise Time	NA	199	42	44	sec.
	Settle Time	NA	214	46.7	51	sec.
	Overshoot	NA	1.1	1.5	2.0	%
	Disturbance	NA	77	83.6	88.7	%
	Rejection					%

achieved.

At 4,059 seconds, both the output air temperature (rT_{ao}) and air flow rate (rF_a) references were increased. The system responded by simultaneously increasing the blower speed (C_{bs}), valve position (C_{vp}), and water heater (C_{wh}) outputs while momentarily increasing the return air damper output (C_{dr}). These actions raised both the output air temperature (T_{ao}) and air flow rates (F_a) with the reference levels being reached in 304 seconds and 387 seconds, respectively. At 4,654 seconds, both the output air (rT_{ao}) and air flow rate (rF_a) references were reduced with the control responding in the reverse of the previous process.

The coordinated control actions from this MIMO controller provided both a faster settling time and better disturbance rejection than the uncoupled PI controllers. The next section provides a detailed comparison of the PI controller and the three MIMO controllers. For a more detailed explanation of the experiments run using the MIMO controllers K_{R1} and K_{R3} , see [1]. Also, an overview of the experiment done with controller K_{R3} appears in [2].

X. PERFORMANCE OF DAS CONTROLLERS

Considering the architectural factors influencing the HVAC controller designs, the performance of the controllers on the experimental system are now contrasted. The settling times of the four DAS controllers in response to step changes in output air temperature and air flow rate are summarized in TABLE X and Fig. 9. These results delineate the increased performance obtained by the coordinated actions of the MIMO controllers.

The reference controller was based upon three (well-tuned) SISO PI controllers that regulated the water supply temperature (T_{ws}), input air temperature (T_{ai}) and output air temperature (T_{ao}), while tracking their respective reference inputs. This design, using the HVAC industries' preferred controller methodology, provided a reference for comparing the performance of the MIMO controllers.

For a step change of 5°C in the discharge air temperature, the system with the SISO PI controllers required 931 seconds to settle at the new level. This was 4.3 times longer than the time required by the fastest MIMO robust controller (K_{R3}). The performance of the PI controller was also evaluated with regard to how well it tracked the reference input in the presence of airflow rate disturbances (ΔF_a). In response to a $0.12 \frac{\text{m}^3}{\text{s}}$ step change in air flow rate (rF_a), the output air temperature (T_{ao}) rose/fell approximately 2°C (96% disturbance

Fig. 8. Controller K_{R2} Experimental Test Results

rejection) before recovering. This was very good, since the best disturbance rejection obtained by a MIMO robust controller (K_{R2}) was only 2.3% better. In response to a change in the air flow rate (blower speed) command (C_{bs}), the air flow rate (F_a) settled to the new value in approximately 5 seconds. However, rapid change in the air flow rate (F_a) disturbed the output air temperature (T_{ao}). While this condition also occurred with the MIMO robust controllers, the reference controller (K_{PI}) was more than five-times slower in returning the output air temperature (T_{ao}) to the reference level.

Controller K_{PI} 's "slow" response time was due in large part to the "low" gains associated with the PI controllers. Recall that these gains were chosen so that the system would remain stable even in the system's highest gain states. Thus, the PI controller was stable throughout the entire operating range of the system. The PI based controller also provided the greatest autonomy and allowed the sharing of water supplies among multiple DASs.

The first MIMO robust controller (K_{R1}), a 3×6 controller, regulated the air flow rate (F_a) and output air temperature

(T_{ao}) to track reference signals. In this design, the water supply temperature (T_{ws}) was externally maintained using a separate PI control and the return air damper command was regarded as a free control variable, allowing the input air temperature (T_{ai}) to be varied. Additionally, as the air flow rate tracked only at low frequency, it was allowed to vary during transitions (to improve performance), but tracked the reference signal at steady-state.

In regulating the output air temperature (T_{ao}), controller K_{R1} (in response to a 5°C step change) was 3.4 times faster in settling to the reference level than the reference (PI) controller and did slightly better in rejecting a ($0.1 \frac{\text{m}^3}{\text{s}}$) step change in the air flow rate. In controlling the air flow rate, K_{R1} had a 214 second settling time, which was 4.6 times slower than that yielded by the fastest MIMO robust controller (K_{R2}).

The second MIMO robust controller (K_{R2}), a 4×7 controller, regulated the input air temperature (T_{ai}), water supply temperature (T_{ws}), air flow rate (F_a) and output air temperature (T_{ao}) to track reference levels. The water supply and output air temperatures tightly tracked the reference inputs while the air flow rate and input air temperatures were allowed to vary in transition, but closely track their references at steady-state. This allowed the controller some freedom for improving both the system's response and disturbance rejection.

In regulating the output air temperature (T_{ao}), controller K_{R2} , had a slightly longer settling time (322s) than did controller K_{R1} (but was still 2.9 times faster than the reference PI controller) and yielded the best disturbance rejection (98.3%) of all the controllers. More importantly, the settling time in response to a step change in air flow rate (46.7 seconds) was a 4.6 times faster than that provided by controller K_{R1} .

The structure of controller K_{R2} was essentially the same as controller K_{R1} , but incorporates control of the water supply temperature into the MIMO controller. While the performance of K_{R2} in response to step changes in air flow rate was markedly better than K_{R1} , this effect was due more to the design performance (weight) specifications than to architectural/structural limitations. Specifically, the performance weight for K_{R1} required twice as much performance (in the performance weight) at DC. This resulted in controller that allowed less overshoot and took longer settle. These results are apparent by comparing K_{R1} and K_{R2} in TABLE X.

The third and final MIMO robust controller design (K_{R3}), also a 4×7 controller, provided the best overall performance of the controller designs. As did controller K_{R2} , this controller regulated the input air temperature (T_{ai}), air flow rate (F_a) and output air temperature (T_{ao}) to track reference levels. However, within this controller, the water heater control output (C_{wh}) was left as a free control variable, allowing the water supply temperature to be varied. The input and output air temperatures tightly tracked their reference inputs¹ while the air flow rate was allowed to vary in transition, but closely tracked its reference at steady-state. This allowed the controller yet more freedom for improving both the system's response

¹The requirement for tracking the input air temperature over "all operating" frequencies was achieved by selecting a larger disturbance weight. Specifically, the disturbance weight used for $w_{Dist}(T_{ao})$ in TABLE V was also used for the $w_{Dist}(T_{ai})$ disturbance weight.

Fig. 9. Comparison of Controller Settling Times

and disturbance rejection. In addition, this controller design used dynamic weights that assumed less model uncertainty at lower frequencies than the other two MIMO robust designs, allowed for a more aggressive controller.

In regulating the output air temperature, K_{R3} provided a settling time of 218 seconds (in response to a 5°C step in output air temperature), the fastest of all of the controllers and 4.3 times faster than the reference PI controller. Additionally, the controller provided a 96.3% disturbance rejection (to a $0.1 \frac{\text{m}^3}{\text{s}}$ step change in the air flow rate). In tracking a step change in the air flow rate, K_{R3} yielded a settle time of 51 seconds. It also provided a 88.7% disturbance rejection (in response to a 5°C step in output air temperature).

In Fig. 9, the settling times of the four controllers are plotted above the controller designations which appear on the horizontal axis. The left and right vertical axes provide the time scales for the output air temperature (black bars) and air flow rate (gray bars), respectively. The sequence of the controllers is related to the architecture (interconnection) of the HVAC system they were intended to control. From left to right, they progress from sharing water supplies (with other DASs) to requiring independent water supplies (for each DAS).

From the preceding, controller K_{R3} clearly out-performed the other controllers. It also required independently controlled (separate) water supplies for each DAS being controlled and differed the most architecturally from today's systems.

A. Factors Affecting Performance

The performance improvements provided by MIMO robust HVAC controllers (over the PI reference controller) were due to several factors. First, the MIMO controllers designed and tested were designed to operate over a limited operating range (since they were linear controllers) and not the entire operating range of the system, where as the PI controllers were designed to be stable over the entire operating range (to conform with current industry standards). Secondly, since the controlled variables were interrelated (e.g., discharge air temperature (T_{ao}), is a function of $-F_w, F_a, T_{wi}, T_{ai}$), the MIMO robust controllers was able to make coordinated control actions

Fig. 10. Interrelationship of Controller Design Objectives/Parameters

using all of the control variables. Such coordinated action provided performance improvements that were not possible using multiple (uncoordinated) SISO controllers. Finally, robust control design methods provided a controller which was insensitive to (limited) variations between the actual HVAC plant and the (nominal) model used during the MIMO design process.

In order to fully make the MIMO controllers applicable in commercial applications, a gain-scheduled robust controller would need to be designed to cover the entire operating range. This task would be an extension to the work presented here and beyond the scope of this article. In this case, the gain-scheduled MIMO robust controller should be compared against a gain-scheduled PI to fairly compare the improvement gained from the MIMO controller.

As shown in Fig. 10, there are four interrelated factors affecting the robust performance provided by a control design. The architecture of the controller must be consistent with that of the HVAC system. Together with the quality (and objectives) of the design implementation, the overall system architecture impacts the performance of the system. A linear controller, controlling the nonlinear HVAC process has a limited operating range in which it may achieve its performance objectives. In a robust controller (on a nonlinear system), the “true” model uncertainty (that not used in covering nonlinearities) allowed for in the design (providing robustness) and the controllers range of operation are interrelated. In general, the greater the (nonlinear) operating range, the less “true” model uncertainty is available. As more and more of the uncertainty is utilized in “covering” nonlinearities, there is less left to cover “true” uncertainties. The less uncertainty available for covering “true” model uncertainty, the less robust the resulting controller design. Similarly, the operating range is affected by the implementation (constraints) of the controller design. As shown in Fig. 10, all of these factors impact the robust performance of the closed-loop HVAC system. The design of HVAC controllers requires consideration of these interrelated factors while seeking a suitable compromise between them.

XI. CONCLUSIONS

The “best” response time of all the HVAC designs considered in this project was provided by controller K_{R3} , a full MIMO robust controller. Response, however, is not the sole criteria for determining the “best” HVAC controller design. The efficiency of the overall system is perhaps the most

important factor considered. Increased efficiency tends to reduce the cost associated with operating the HVAC system. However, the economics associated with an HVAC system are determined by the initial cost of building the system, as well as the operating costs (efficiency). While increased performance may increase efficiency, the associated cost of doing so may not always be economically justified. The various HVAC controller designs considered by this project provide different performance merits. Many of these merits are closely related to the architecture of the HVAC system for which they were designed to control.

The PI controller, consisting of separate SISO (PI) controllers, is well suited to today’s HVAC systems, which have water supplies supporting multiple DASs. The system response is “slow,” but the efficiency afforded by sharing air and water supplies is economically appealing.

Controller K_{R3} , a “full” MIMO robust design, responded the fastest of all the controllers (a step in T_{ao} settles in 218 second). However, this design requires independent (separate) water supplies for each DAS controlled and it differs the most architecturally from today’s systems. The cost associated with building a system with separate water supplies for each DAS may be economically hard to justify.

While more constrained, controller K_{R2} provides much of the performance (response) of controller K_{R3} (a step in T_{ao} settles in 322 second). This controller design is compatible with HVAC systems where the DASs share a common water supply. It offers a good compromise between performance and the economics of a shared water supply.

The controllers K_{R2} and K_{R1} operated with an externally controlled water supply (boiler) and provided a significant increase in performance over the reference system. The performance differences (in tracking reference airflow inputs (rF_a)) exhibited by these controllers were due primarily to differences in the (performance) weighting functions used in their design, not differences in their architecture. These two controllers would be the most practical for current HVAC architectures.

The experimental results demonstrate that the application of robust MIMO controls to an HVAC system offers a dramatic (more than 3 times) improvement in performance over current PI based HVAC controllers. Furthermore, increased performance may be realized without the increased cost associated with separate water supplies (for each DAS controlled), that are required with a “full MIMO” controller.

This article was not intended to address the various PI designs (i.e. only a well tuned industry standard PI controller was used), but rather exemplify the design and implementation of MIMO robust controllers for various HVAC system architectures (one of which represents a major shift in the current design paradigm) and to demonstrate the improvement that would be expected on a commercial HVAC system. This article (based on the research in [1]) is one of the first to approach HVAC controller design from a multivariable point of view.

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