Sets

1. Use set builder notation to give a description of this set: 
\{ 0,3,6,9,12 \}

2. Determine whether each of these statements is true or false:
   (a) \( x \in \{ x \} \)
   (b) \( \{ x \} \subseteq \{ x \} \)
   (c) \( \{ x \} \in \{ x \} \)

3. Use a Venn diagram to illustrate the set of all months of the year whose names do not contain the letter \( r \) in the set of all months of the year.

4. Suppose that A, B, and C are sets such that \( A \subseteq B \) and \( B \subseteq C \). Use a Venn diagram to show that \( A \subseteq C \).

5. Let \( A = \{ a, b, c \} \), \( B = \{ x, y \} \), and \( C = \{ 0, 1 \} \). Show the contents of the set: 
\( C \times B \times A \).

6. Let \( A = \{ 1,2,3,4,5 \} \) and \( B = \{ 0,3,6 \} \). Show the contents of each of the following:
   (a) \( A \cup B \)
   (b) \( A \cap B \)
   (c) \( A - B \)
   (d) \( B - A \)

7. Let \( A = \{ 7,8 \} \) and let \( B = \{ a,b,c \} \).
   (a) What is the cardinality of \( A \): \( | A | \)?
   (b) What is the intersection of \( A \) and \( B \): \( A \cap B \)?
   (c) What is the powerset of \( A \): \( P(A) \)?
(d) What is the cardinality of the powerset of A: $|P(A)|$?

(e) What is the powerset of B: $P(B)$?

(f) What is the Cartesian product of B and A: $B \times A$?

(g) What is the cardinality of the Cartesian product of B and A: $|B \times A|$?

8. What can you say about the sets A and B if you know the following? (The first one is done for you.)

(a) $A \cup B = A$ (We know that $B \subseteq A$.)

(b) $A \cap B = A$

(c) $A - B = A$

(d) $A \cap B = B \cap A$

(e) $A - B = B - A$

9. Suppose that the universal set is $U = \{1,2,3,4,5,6,7,8,9,10\}$. Express each of these sets with bit strings where the $i$th bit in the string is 1 if $i$ is in the set and 0 otherwise.

(a) $\{3, 4, 5\}$

(b) $\{1, 3, 6, 10\}$

(c) $\{2, 3, 4, 7, 8, 9\}$

**Functions**

10. For the following functions, state whether $f$ is a function from $\mathbb{Z}$ to $\mathbb{R}$:

(a) $f(n) = \pm n$

(b) $f(n) = \sqrt{n^2 + 1}$

11. What are the domain and range of the function that assigns to each positive integer its largest decimal digit?

12. Is the following function from $\mathbb{Z}$ to $\mathbb{Z}$ one-to-one?

$f(n) = n^2 + 1$