The product rule

- If there are \( n_1 \) ways of doing one task, and for each of way of doing the first task there are \( n_2 \) ways of doing a second task, then there are \( n_1n_2 \) ways of performing both tasks.

Example

- You have 6 pairs of pants and 10 shirts. How many different outfits does this give?

Relation to cartesian products

- The cartesian product of sets A and B is denoted by \( A \times B \) and is defined as:
  \[
  \{ (a,b) \mid a \in A \text{ and } b \in B \}
  \]
- Fact: \( |A \times B| = |A| \times |B| \)

Product rule

- Colorado assigns license plates numbers as a-b-c x-y-z, where a,b,c are digits and x,y,z are letters. How many license plates numbers are possible?

More examples

- How many binary numbers with 7 digits are there?
- How many functions are there from a set with \( m \) elements to a set with \( n \) elements?
- How many one-to-one functions are there from a set with \( m \) elements to a set with \( n \) elements?
More examples

Use the product rule to show that the number of different subsets of a finite set \( S \) is \( 2^{\|S\|} \).

A woman has decided to shop at one store today, either in the north end of town or the south end of town. If she visits the north end, she will shop at one of three stores. If she visits the south end of town then she will shop at one of two stores. How many ways could she end up shopping?

Example

A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 1 possible projects. No project is on more than one list. How many possible are there to choose from?

Product rule and cartesian products

\[ |A_1 \times A_2 \times \ldots \times A_n| = |A_1| \times |A_2| \times \ldots \times |A_n| \]

The sum rule

If a task can be done either in one of \( n_1 \) ways or in one of \( n_2 \) ways, and none of the \( n_1 \) ways is the same as the \( n_2 \) ways, then there are \( n_1 + n_2 \) ways to do the task.

This is a statement about set theory: if two sets \( A \) and \( B \) are disjoint then
\[ |A \cup B| = |A| + |B| \]

More complicated counting

Suppose you need to pick a password that has length 6-8 characters, where each character is an uppercase letter or a digit, and each password must contain at least one digit. How many possible passwords are there?
The inclusion exclusion principle

- How many bit strings of length eight either start with a 1 or end with a 0?

In set theory terms:

\[ A \cup B = |A| + |B| - |A \cap B| \]

The pigeonhole principle

- If \( k \) is a positive integer and \( k+1 \) or more objects are placed into \( k \) boxes, then there is at least one box containing two or more objects.

Examples

- In a group of 367 people, there must be at least two with the same birthday

  - A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A guy takes socks out at random in the dark.
    - How many socks must he take out to be sure that he has at least two socks of the same color?
    - How many socks must he take out to be sure that he has at least two black socks?

Examples

- Show that if five integers are selected from the first eight positive integers, there must be a pair of these with a sum equal to 9.

Permutations

- In a family of 5, how many ways can we arrange the members of the family in a line for a photograph?
Permutations

- A permutation of a set of distinct objects is an ordered arrangement of these objects.