1. For the following functions find the least integer $d$ such that $f(x)$ is $O(x^d)$, and formally show that $f(x)$ is $O(x^d)$ for the value of $d$ stated.

   (a) $f(x) = x^2 + x^3 \log(x)$  
   (b) $f(x) = x^8 + 10x^4 + x + 1$  
   (c) $f(x) = (x^8 + 10x^4)(x^2 + x) + x^9$  
   (d) $f(x) = \frac{\log(x)}{x}$  
   (e) $f(x) = 2^x + x^2$

2. **Analysis of your programming assignment**

   In your first programming assignment you populated a database of movies rated by customers.

   (a) Describe your method for populating the database and estimate a tight big-O bound for the process. Assume the database contains $n$ customers, and each customer has rated $m$ movies at the most.

   (b) Describe your implementation of the method that computes the intersection of the movies rated by two customers and provide a tight big-O bound for its running time on a database that contains $n$ customers, where each customers has rated $m$ movies at the most.

3. What is the lowest big-O bound for the running time of the following snippet of code?

   ```c
   int i = 1;  
   int j = 0;  
   while (i < n) {  
      j++;  
      i = i * 10;  
   }
   ```

4. Give a big-O estimate of the product of the first $n$ odd positive integers, i.e. a bound on the value of the product $1 \cdot 3 \cdot 5 \ldots (2n - 1)$ (not a bound on a method for computing the product). Your bound should be a simple function like $n^d$, as opposed to a product, so we can tell its rate of growth.

5. Show that $2^n$ is $O(3^n)$ but that $3^n$ is not $O(2^n)$.

6. Show that if $f(n)$ is $O(n)$ then $f(n)$ is $O(n^2)$. Extra credit: how would you generalize this result? (hint: transitivity)