

Logic

- True and False
- AND, OR, Exclusive OR
- Truth Tables
- Complex Expressions
- Implication; IF...THEN
- Logic Puzzles

TRUE and FALSE

- The core values of logic
 - Often represented in computers as 0 (false) and 1 (true) or 0 (false) and all other numbers (true)
 - Can be represented in a single bit (hence, a byte could store 8 true-false values)
 - Expressions evaluate to true or false:
 - $(7 > 8)$ is false
 - $(8+1 == 9)$ is true
 - $(X \geq Y)$ may be true or false, depending on the values of X and Y

AND, OR, XOR (and NOT)

- x AND y is true when both x and y are true
 - False if either one is false
- x OR y is true when either x or y are true
 - True if both are true
 - False only if both are false
- x XOR y is true when x is true, or y is true, but not both
 - False if both have the same truth value
- NOT x is true when x is false

Truth Tables

X	Y	X and Y
F	F	F
F	T	F
T	F	F
T	T	T

X	Y	X xor Y
F	F	F
F	T	T
T	F	T
T	T	F

Complex Expressions

$(X \text{ and } Y) \text{ or } (Y \text{ and } Z)$

$Y \text{ or } (X \text{ and } Z)$

$Y \text{ and } (X \text{ or } Z)$

$(X \text{ and } (\text{not } Y)) \text{ or } (Y \text{ and } (\text{not } X))$

Implication

- $A \rightarrow B$ is true if
 - A is true and B is true, or
 - A is false
- $A \rightarrow B$ is also written as “if A then B ”
- For example:
 - If $((x \geq 0) \text{ and } (x < 10))$ then $(x^2 < 100)$ is TRUE
- Yes, this can lead to some “nonsense”-sounding clauses:
 - If $(4 < 3)$ then $(75 > 100)$ is TRUE
- Even some theological quandries
 - If $(1 < 0)$ then god does not exist is also TRUE
 - Note, if you make that “if $(1 > 0) \dots$ ” we can’t tell!

Logic Puzzles

- Fun, games, GRE scores!
- Basic idea is not to “prove” anything, but to think through them logically

Example

- Mr. Red, Ms. Green, and Mrs. Blue each live in a different one of the three houses numbered 800, 802, and 804.
- Each of them wears a shirt with the color that matches another person's name. All three shirt colors are used.
- The person in house 802 is not wearing a Green shirt.
- The person in house 800 is Ms. Green.
- Mrs. Blue is wearing a red shirt.
- Who lives where and wears which shirt?

Start with simple things

- Mrs. Blue wears the red shirt
 - That means, Ms. Green must wear blue
 - And Mr. Red must wear Green
- Ms. Green is in house 800
 - Mr. Red cannot live in 802 (Green Shirt), so he must be in 804, and Mrs. Blue must be in 802.