

Rules of Inference

Rule of Inference	Tautology	Name
$\frac{p}{\therefore p \vee q}$	$p \longrightarrow (p \vee q)$	Addition
$\frac{p \wedge q}{\therefore p}$	$(p \wedge q) \longrightarrow p$	Simplification
$\frac{p \quad p \rightarrow q}{\therefore q}$	$(p \wedge (p \rightarrow q)) \longrightarrow q$	Modus ponens
$\frac{\sim q \quad p \rightarrow q}{\therefore \sim p}$	$(\sim q \wedge (p \rightarrow q)) \longrightarrow \sim p$	Modus tollens
$\frac{p \rightarrow q \quad q \rightarrow r}{\therefore p \rightarrow r}$	$((p \rightarrow q) \wedge (q \rightarrow r)) \longrightarrow (p \rightarrow r)$	Hypothetical syllogism
$\frac{p \vee q \quad \sim p}{\therefore q}$	$((p \vee q) \wedge \sim p) \longrightarrow q$	Disjunctive syllogism
$\frac{p \rightarrow r \quad q \rightarrow r}{\therefore (p \vee q) \rightarrow r}$	$((p \rightarrow r) \wedge (q \rightarrow r)) \longrightarrow ((p \vee q) \rightarrow r)$	Rule for proof by cases.