

Rules of Inference

1. **Addition ROI:** $p \rightarrow (p \vee q)$
 2. **Simplification ROI:** $(p \wedge q) \rightarrow p$
 3. **Conjunction ROI:** $((p) \wedge (q)) \rightarrow (p \wedge q)$
 4. **Modus Ponens:** $(p \wedge (p \rightarrow q)) \rightarrow q$
 5. **Modus Tollens:** $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$
 6. **Hypothetical Syllogism:** $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$
 7. **Disjunctive Syllogism:** $((p \vee q) \wedge \neg p) \rightarrow q$
 8. **Resolution:** $((p \vee q) \vee (\neg p \vee r)) \rightarrow (q \vee r)$
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Rules of Inference for Quantified Statements:

1. **Universal Instantiation ROI:** $\forall x P(x) \therefore P(c)$
 2. **Universal Generalization ROI:** $P(c)$ for an arbitrary $c \therefore \forall x P(x)$
 3. **Existential Instantiation ROI:** $\exists x P(x) \therefore P(c)$ for some element c
 4. **Existential Generalization ROI:** $P(c)$ for some element $c \therefore \exists x P(x)$
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Contrapositive: $(\neg q \rightarrow \neg p) \equiv (p \rightarrow q)$

Contradiction: $((p \rightarrow q) \wedge (\neg q)) \rightarrow (\neg p)$

De Morgan's:

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$