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Yet Even More Proofs

Proof techniques

Strategies

Examples

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Example I

- Given: ***m*** is an even integer and ***n*** is an odd integer

- Prove: the sum of ***n*** and ***m*** is odd

Example I, Formal Proof

	Step	Reason
1.	***n*** is odd	Premise
2.		
3.		
4.		
5.		
6.		
7.		

Example I, Comments

- Notice how we defined ***n*** and ***m***:
  - $\exists k \in Z \;\; n = 2k+1$
  - $\exists j \in Z \;\; m = 2j$

- Why didn’t we just use ***k*** for both of them?

Example II

- Given: n^2 is an odd integer
- Prove: n is an odd integer
- Recall that we proved the reverse of this in a previous class, using a **direct proof**
- This proof requires

Example II, Formal Proof

	Step	Reason
1.	***n*** is not odd	¬Conclusion, assumption
2.	***n*** is even	If not odd
3.		
4.		
5.		
6.		
7.		

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Example II, Conclusion

Since we have proved the contrapositive:

We have also proven the original hypothesis:

Example III

(Adapted from Kolman, Busby, & Ross, Discrete Mathematical Structures, 6th ed., Pearson/PrenticeHall (2004))

- Given: ***m*** and ***n*** are integers

$\leftrightarrow$ if and only if

- Prove: $n^2 = m^2 \leftrightarrow n = m \vee n = -m$
- This is a proof of **equivalence**
- Must prove *both* ways **(*two* proofs)**:

Example III, part one

$$(n = m \vee n = -m) \rightarrow (n^2 = m^2)$$

- $((q \vee r) \rightarrow p)$ is the form of the proposition to be proven, where
  - p is $n^2 = m^2$
  - q is $n = m$
  - r is $n = -m$
- We have two cases to prove:
  -
  -
- Each case can be done as a

Example III, part one

$(n = m \lor n = -m) \rightarrow (n^2 = m^2)$

- Case 1: $(n = m) \rightarrow (n^2 = m^2)$
  - Assume ***q*** is true; so $n = m$

Example III, part one (formal)

$(n = m \vee n = -m) \rightarrow (n^2 = m^2)$

Case 1: $(n = m) \rightarrow (n^2 = m^2)$

	Step	Reason
1.	$n = m$	Premise
2.		
3.		

Example III, part one

$(n = m \vee n = -m) \rightarrow (n^2 = m^2)$

- Case 2: $(n = -m) \rightarrow (n^2 = m^2)$
  - Assume ***r*** is true; so
  - By squaring each side of the equation *(algebra)*, we have
  - So

Example III, part one (formal)

$(n = m \vee n = -m) \rightarrow (n^2 = m^2)$

Case 2: $(n = -m) \rightarrow (n^2 = m^2)$

	Step	Reason
1.	$n = -m$	Premise
2.		
3.		

Example III, part two

$$(n^2 = m^2) \rightarrow (n = m \vee n = -m)$$

- $(p \rightarrow (q \vee r))$ is the form of the proposition to be proven, where
  - p is $n^2 = m^2$
  - q is $n = m$
  - r is $n = -m$
- This can be done as an
- This proof is *not* so straightforward

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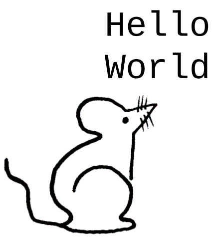
Example III, part two

$$(n^2 = m^2) \rightarrow (n = m \vee n = -m)$$

- $n^2 = m^2$ (Premise)
-
-
-
-
-
-



Example III, part two, cont.

$(n^2 = m^2) \rightarrow (n = m \vee n = -m)$

- To show: $s \rightarrow (q \vee r)$, by showing

- Contrapositive: $\neg(q \vee r) \rightarrow \neg s$

- By DeMorgan's Laws, it is sufficient to show: $((\neg q) \wedge (\neg r)) \rightarrow (\neg s)$

Example III, part two, cont.

$$(n^2 = m^2) \rightarrow (n = m \vee n = -m)$$

- To show: $((\neg q) \wedge (\neg r)) \rightarrow (\neg s)$

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Example III, part two, cont.

$$(n^2 = m^2) \rightarrow (n = m \lor n = -m)$$

- To show: $((\neg q) \land (\neg r)) \rightarrow (\neg s)$, cont.

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Example III, part two, concluded

$$(n^2 = m^2) \rightarrow (n = m \vee n = -m)$$

- $((\neg q) \wedge (\neg r)) \rightarrow (n+m)(n-m) \neq 0$ (last slide)

Example III, concluded

- In part one, we proved
  - $(n = m \vee n = -m) \rightarrow (n^2 = m^2)$
- In part two, we proved that
  - $(n^2 = m^2) \rightarrow (n = m \vee n = -m)$
- Hence,