
CS 161

HOMEWORK 1

Due in class - 3/14 for Asa's section, 3/15 for Nayeem's

Proof by mathematical induction

You must show your work: the formula you are using, then substitute the numbers into the formula, then calculate the answer. Hand in to your instructor during lecture.

Prove the following statements using induction. Each proof needs to be complete, including the basis step, and induction step. Each step in the proof needs to be justified (except when it follows by simple algebra). If you are using strong induction, state so explicitly.

1. Let $P(n)$ be the statement that $1^3 + 2^3 + \cdots + n^3 = [n(n+1)/2]^2$ for the positive integer n .
 - (a) What is the statement $P(1)$?
 - (b) Show that $P(1)$ is true (basis step).
 - (c) What is the induction hypothesis?
 - (d) What do you need to prove in the inductive step?
 - (e) Complete the inductive step.
 - (f) Explain why these steps show that this formula is true for every positive integer n .
2. Prove that $1 \cdot 1! + 2 \cdot 2! + \cdots + n \cdot n! = (n+1)! - 1$ for every positive integer n .
3. Prove that for every positive integer n ,

$$1^2 + 3^2 + 5^2 + \cdots + (2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$$

4. For which nonnegative integers n is $n^2 \leq n!$? Prove your answer.
5. Prove that the sum of the first n even positive integers is $n(n+1)$.
6. Prove that 3 divides $n^3 + 2n$ for every positive integer n .
7. Show that you can pay any amount of money greater than \$5 using only two-dollar and five-dollar bills.
8. What is wrong with this "proof": For every positive integer n , $\sum_{i=1}^n i = (n+1/2)^2/2$.

Basis step: The formula is true for $n = 1$.

Inductive step: Suppose that $\sum_{i=1}^n i = (n+1/2)^2/2$. Then $\sum_{i=1}^{n+1} i = \sum_{i=1}^n i + (n+1)$.

By the induction hypothesis,

$\sum_{i=1}^{n+1} i = (n+1/2)^2/2 + (n+1) = [(n+1) + 1/2]^2/2$, completing the inductive step.