
CS 161

HOMWORK 2

Due Due in class - 3/28 for Asa's section; 3/29 for Nayeem's

You must explain your answer (which counting principles are used at each step and why). Show the formulas you are using, then substitute the numbers into the formula, then calculate the answer. Hand in to your instructor during lecture.

1. How many people are needed to guarantee that at least two were born on the same day of the week (Sunday, Monday, Tuesday, etc.) and in the same month?
2. How many ways are there to select 12 countries in the United Nations to serve on a council if 3 are selected from a block of 40, 4 are selected from a block of 55, and others are selected from the remaining 65 countries?
3. How many strings are there of four lowercase letters that have the letter x in them?
4. There are four major routes from Boston to Detroit and six from Detroit to Los Angeles. How many major routes are there from Boston to Los Angeles via Detroit?
5. How many license plates can be made using either three digits followed by three letters or three letters followed by three digits?
6. How many subsets of a set with 100 elements have more than one element? (i.e. at least 2 elements)
7. How many initials that contain three letters and begin with a C or a D are there? (case is not relevant here).
8. Let d be a positive integer. Show that among any group of $d + 1$ (not necessarily consecutive) integers there are two with exactly the same remainder when they are divided by d (hint: use the pigeonhole principle).
9. Let S and T be finite sets such that $|S| > |T|$, and let $f : S \rightarrow T$ be a function. Show that there exist elements $s_1, s_2 \in S$ such that $f(s_1) = f(s_2)$. This shows that f is not one-to-one.
10. Let n be a positive integer ($n \geq 1$). How many functions f are there from the set $\{1, \dots, n\}$, to the set $\{0, 1\}$ such that:
 - (a) f is one to one.
 - (b) f assigns 0 to both 1 and n .
 - (c) f assigns 1 to exactly one number between 1 and n .