Counting: Basics

Rosen, Chapter 6.1

Counting: the product rule

- If there are n_1 ways of doing one task, and for each of way of doing the first task there are n_2 ways of doing a second task, then there are $n_1 n_2$ ways of performing both tasks.
- Example:
  - You have 6 pairs of pants and 10 shirts. How many different outfits does this give?

Relation to Cartesian products

- The Cartesian product of sets A and B is denoted by ***A x B*** and is defined as:
  A x B = { (a,b) | a ∈ A and b ∈ B}
- | **A x B** | = | **A** | * | **B** |

Product rule

- Colorado assigns license plates numbers as a-b-c x-y-z, where a,b,c are digits and x,y,z are letters. How many license plates numbers are possible?

More examples

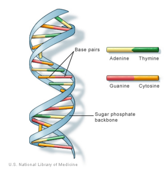
- How many binary numbers with 7 digits are there?
- How many functions are there from a set with m elements to a set with n elements?
- How many one-to-one functions are there from a set with m elements to a set with n elements?

More examples

- Use the product rule to show that the number of different subsets of a finite set S is $2^{|S|}$
- Product rule and Cartesian products:
  - $|A_1 \times A_2 \times \ldots \times A_n| = |A_1| \times |A_2| \times \ldots \times |A_n|$

DNA and proteins

- DNA is a long chain that contains one of four nucleotides (A,C,G,T). DNA can code for proteins that are chains of amino acids. There are 20 amino acids. How many nucleotides does it take to code for a single amino acid?



Sum Rule

- X has decided to shop at a single store, either in the north end of town or the south end of town. If X visits the north, X will shop at one of three stores. If X visits the south, then X will shop at one of two stores. How many ways could X end up shopping?

The Sum Rule

- If a task can be done either in one of n_1 ways or in one of n_2 ways, and none of the n_1 ways is the same as the n_2 ways, then there are $n_1 + n_2$ ways to do the task.
- This is a statement about set theory: if two sets A and B are disjoint then

$$|A \cup B| = |A| + |B|$$

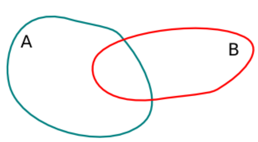
Example

- A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 4 possible projects. No project is on more than one list. How many possible projects are there to choose from?

The inclusion exclusion principle

- A more general statement than the sum rule:

$$|A \cup B| = |A| + |B| - |A \cap B|$$



Example

- Suppose you need to pick a password that has length 6-8 characters, where each character is an uppercase letter or a digit, and each password must contain at least one digit. How many possible passwords are there?
- (Recall: $|A \cup B| = |A| + |B| - |A \cap B|$)

The inclusion exclusion principle

- How many bit strings of length eight start with a 1 **or** end with 00?

1 - - - - - - -   how many?
- - - - - - 0 0   how many?

if I add these
how many have I now counted twice?

The pigeonhole principle

- If k is a positive integer and k+1 or more objects are placed into k boxes, then there is at least one box containing two or more objects.

Examples

- In a group of 367 people, there must be at least two with the same birthday
- A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A guy takes socks out at random in the dark.
  - How many socks must he take out to be sure that he has at least two socks of the same color?
  - How many socks must he take out to be sure that he has at least two black socks?

Examples

- Show that if five different digits between 1 and 8 are selected, there must be at least one pair of these with a sum equal to 9.
- ask yourself: what are the pigeon holes?
  what are the pigeons?