Pre-test practice recitation

CS161

July 13, 2016

1 General Instructions

Refer to the PDF in the class code (july-11/resources):
Inductive Proof Examples

2 Counting

1. How many positive integers less than 1000
   (a) have exactly three digits?
   (b) have an odd number of digits?
   (c) have at least one digit equal to 9?
   (d) have no odd digits?
   (e) have two consecutive digits equal to 5?
   (f) are palindromes?
2. How many ways are there to choose a dozen donuts from 20 varieties
   (a) if there are no two donuts of the same variety?
   (b) if all donuts are of the same variety?
   (c) if there are no restrictions?
   (d) if there are at least two varieties?
   (e) if there must be at least six blueberry donuts?
3. In how many different ways can five elements be selected in order from a set with three elements when repetition is allowed?
4. How many strings of six letters are there?
   (a) if only lowercase is allowed?
   (b) if any case is allowed?
5. In how many ways can you stack 7 different books, so that a specific book B is on the third place?

6. In how many ways can you take 3 marbles out of a box with 15 different marbles?

7. In how many ways can you take 5 cards, with at least 2 aces, out of a deck of 52 cards?

8. Find n if

   (a) ${}^nP_2 = 110$

   (b) ${}^nP_n = 5040$

   (c) ${}^nP_4 = 12 * {}^nP_2$

9. Find n if

   (a) $\binom{n}{2} = 45$

   (b) $\binom{n}{3} = {}^nP_2$

   (c) $\binom{n}{5} = \binom{n}{2}$

10. If the numbers from 1 to 1000 are written out on a piece of paper, how many 9's are on that paper?

3 Proofs

1. Prove that for all positive integers, n:

$$\frac{2}{3} + \frac{2}{9} + \frac{2}{27} + ... + \frac{2}{3^n} = 1 - \frac{1}{3^n}$$

2. Which amounts of postage can you make using 5 and 9 cent stamps?

3. Find $f(1)$, $f(2)$, $f(3)$, and $f(4)$ if $f(n)$ is defined recursively by $f(0) = 1$ and for $n = 0, 1, 2, ...$

   (a) $f(n+1) = f(n) + 2$

   (b) $f(n+1) = 3f(n)$

   (c) $f(n+1) = 2^{f(n)}$

   (d) $f(n+1) = f(n)^2 + f(n) + 1$

4. Let $P(n)$ be the statement that

$$1 + \frac{1}{4} + \frac{1}{9} + ... + \frac{1}{n^2} < 2 - \frac{1}{n}$$

   where n is an integer greater than 1.

   (a) What is the statement $P(2)$?

   (b) Show that $P(2)$ is true, completing the basis step of the proof.

(c) What is the *inductive hypothesis*?

(d) What do you need to prove in the inductive step?

(e) Complete the inductive step.

(f) Explain why these steps show that this inequality is true for all n where n is an integer greater than 1.

5. Prove by induction that:

$$1 + 3 + 5 + ... + (2n - 1) = n^2$$