

Chapter 23 Sorting

CS1: Java Programming
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Objectives

- ◆ To study and analyze time complexity of various sorting algorithms (§§23.2–23.7).
- ◆ To design, implement, and analyze insertion sort (§23.2).
- ◆ To design, implement, and analyze bubble sort (§23.3).
- ◆ To design, implement, and analyze merge sort (§23.4).



Why study sorting?

Sorting is a classic subject in computer science. There are three reasons for studying sorting algorithms.

- First, sorting algorithms illustrate many creative approaches to problem solving and these approaches can be applied to solve other problems.
- Second, sorting algorithms are good for practicing fundamental programming techniques using selection statements, loops, methods, and arrays.
- Third, sorting algorithms are excellent examples to demonstrate algorithm performance.



What data to sort?

The data to be sorted might be integers, doubles, characters, or objects. §7.8, “Sorting Arrays,” presented selection sort and insertion sort for numeric values. The selection sort algorithm was extended to sort an array of objects in §11.5.7, “Example: Sorting an Array of Objects.” The Java API contains several overloaded sort methods for sorting primitive type values and objects in the `java.util.Arrays` and `java.util.Collections` class. For simplicity, this section assumes:

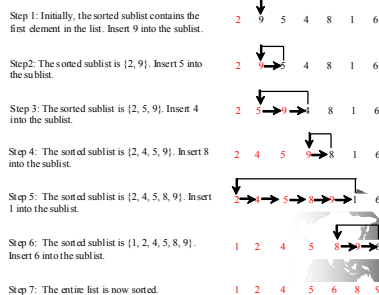
- ◆ data to be sorted are integers,
- ◆ data are sorted in ascending order, and
- ◆ data are stored in an array. The programs can be easily modified to sort other types of data, to sort in descending order, or to sort data in an `ArrayList` or a `LinkedList`.



Insertion Sort

```
int[] myList = {2, 9, 5, 4, 8, 1, 6}; // Unsorted
```

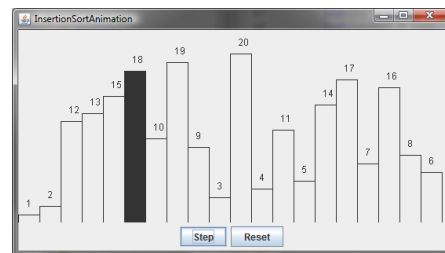
The insertion sort algorithm sorts a list of values by repeatedly inserting an unsorted element into a sorted sublist until the whole list is sorted.



animation

Insertion Sort Animation

<http://www.cs.armstrong.edu/liang/animation/web/InsertionSort.html>



animation

Insertion Sort

```
int[] myList = {2, 9, 5, 4, 8, 1, 6}; // Unsorted
```

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How to Insert?

The insertion sort algorithm sorts a list of values by repeatedly inserting an unsorted element into a sorted sublist until the whole list is sorted.

```
[0] [1] [2] [3] [4] [5] [6]
list 2 5 9 4
```

Step 1: Save 4 to a temporary variable currentElement

```
[0] [1] [2] [3] [4] [5] [6]
list 2 5 9
```

Step 2: Move list[2] to list[3]

```
[0] [1] [2] [3] [4] [5] [6]
list 2 5 9
```

Step 3: Move list[1] to list[2]

```
[0] [1] [2] [3] [4] [5] [6]
list 2 4 5 9
```

Step 4: Assign currentElement to list[1]

```
[0] [1] [2] [3] [4] [5] [6]
list 2 4 5 9
```

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From Idea to Solution

```
for (int i = 1; i < list.length; i++) {
    insert list[i] into a sorted sublist list[0..i-1] so that
    list[0..i] is sorted
}
```

```
list[0]

list[0] list[1]

list[0] list[1] list[2]

list[0] list[1] list[2] list[3]

list[0] list[1] list[2] list[3] ...
```

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From Idea to Solution

```
for (int i = 1; i < list.length; i++) {
    insert list[i] into a sorted sublist list[0..i-1] so that
    list[0..i] is sorted
}
```

Expand

```
double currentElement = list[i];
int k;
for (k = i - 1; k >= 0 && list[k] > currentElement; k--) {
    list[k + 1] = list[k];
}
// Insert the current element into list[k + 1]
list[k + 1] = currentElement;
```

InsertSort Run

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Bubble Sort

| | | | | |
|-------------|-------------|-------------|-------------|-------------|
| 2 9 5 4 8 1 | 2 5 4 8 1 9 | 2 4 5 1 8 9 | 2 4 1 5 8 9 | 1 2 4 5 8 9 |
| 2 5 4 8 1 | 2 4 5 8 1 9 | 2 4 5 1 8 9 | 2 1 4 5 8 9 | |
| 2 5 4 8 1 | 2 4 5 8 1 9 | 2 4 1 5 8 9 | | |
| 2 5 4 8 1 | 2 4 5 1 8 9 | | | |
| 2 5 4 8 1 | | | | |

(a) 1st pass (b) 2nd pass (c) 3rd pass (d) 4th pass (e) 5th pass

Bubble sort time: $O(n^2)$

$$(n-1) + (n-2) + \dots + 2 + 1 = \frac{n^2}{2} - \frac{n}{2}$$

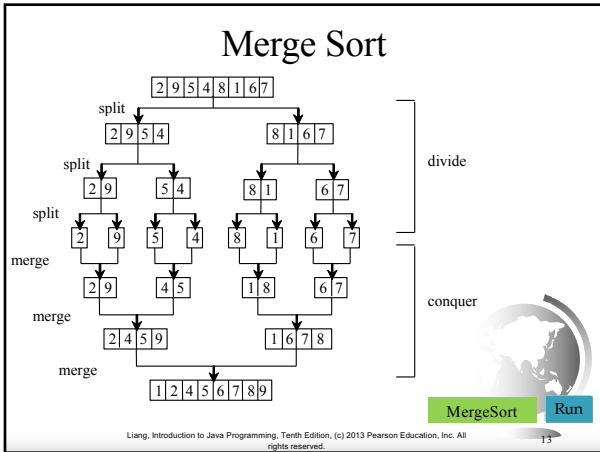
BubbleSort Run

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Bubble Sort Animation

<http://www.cs.armstrong.edu/liang/animation/web/BubbleSort.html>

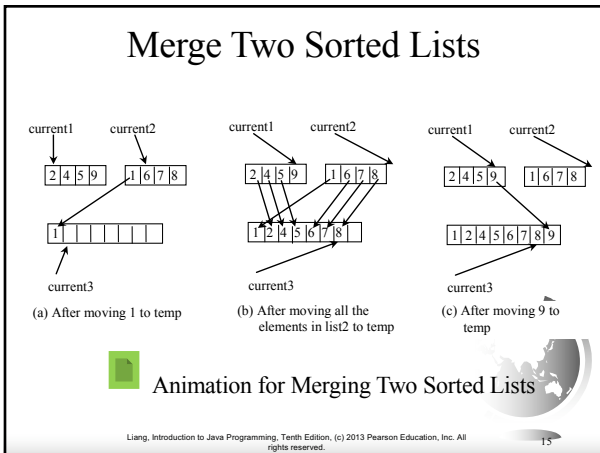
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Merge Sort

```
mergeSort(list):
    firstHalf = mergeSort(firstHalf);
    secondHalf = mergeSort(secondHalf);
    list = merge(firstHalf, secondHalf);
```

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Merge Sort Time

Let $T(n)$ denote the time required for sorting an array of n elements using merge sort. Without loss of generality, assume n is a power of 2. The merge sort algorithm splits the array into two subarrays, sorts the subarrays using the same algorithm recursively, and then merges the subarrays. So,

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + \text{mergetime}$$

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + O(n)$$

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Merge Sort Time

The first $T(n/2)$ is the time for sorting the first half of the array and the second $T(n/2)$ is the time for sorting the second half. To merge two subarrays, it takes at most $n-1$ comparisons to compare the elements from the two subarrays and n moves to move elements to the temporary array. So, the total time is $2n-1$. Therefore,

$$T(n) = 2T\left(\frac{n}{2}\right) + 2n - 1 = 2(2T\left(\frac{n}{4}\right) + 2\frac{n}{2} - 1) + 2n - 1 = 2^2T\left(\frac{n}{2^2}\right) + 2n - 2 + 2n - 1$$

$$= 2^k T\left(\frac{n}{2^k}\right) + 2n - 2^{k-1} + \dots + 2n - 2 + 2n - 1$$

$$= 2^{\log_2 n} T\left(\frac{n}{2^{\log_2 n}}\right) + 2n - 2^{\log_2 n - 1} + \dots + 2n - 2 + 2n - 1$$

$$= n + 2n \log_2 n - 2^{\log_2 n} + 1 = 2n \log_2 n + 1 = O(n \log n)$$

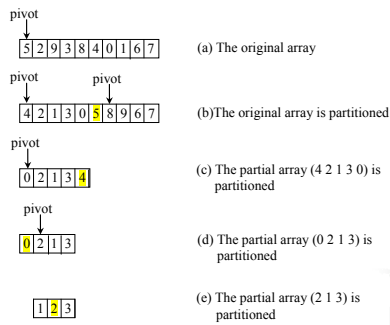
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Quick Sort

Quick sort, developed by C. A. R. Hoare (1962), works as follows: The algorithm selects an element, called the *pivot*, in the array. Divide the array into two parts such that all the elements in the first part are less than or equal to the pivot and all the elements in the second part are greater than the pivot. Recursively apply the quick sort algorithm to the first part and then the second part.

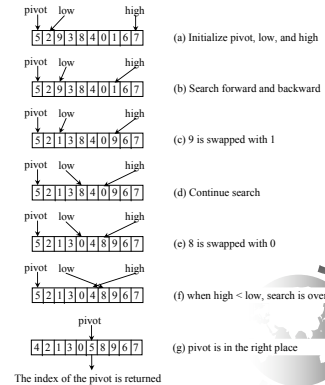
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Quick Sort



Partition

Animation for partition



QuickSort Run

Quick Sort Time

To partition an array of n elements, it takes $n-1$ comparisons and n moves in the worst case. So, the time required for partition is $O(n)$.

Worst-Case Time

In the worst case, each time the pivot divides the array into one big subarray with the other empty. The size of the big subarray is one less than the one before divided. The algorithm requires $O(n^2)$ time:

$$(n-1) + (n-2) + \dots + 2 + 1 = O(n^2)$$

Best-Case Time

In the best case, each time the pivot divides the array into two parts of about the same size. Let $T(n)$ denote the time required for sorting an array of n elements using quick sort. So,

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + n = O(n \log n)$$

Average-Case Time

On the average, each time the pivot will not divide the array into two parts of the same size nor one empty part. Statistically, the sizes of the two parts are very close. So the average time is $O(n \log n)$. The exact average-case analysis is beyond the scope of this book.

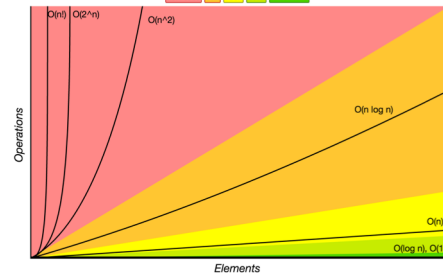
Computational Complexity (Big O)

- ◆ $T(n)=O(1)$ // constant time
- ◆ $T(n)=O(\log n)$ // logarithmic
- ◆ $T(n)=O(n)$ // linear
- ◆ $T(n)=O(n \log n)$ // linearithmic
- ◆ $T(n)=O(n^2)$ // quadratic
- ◆ $T(n)=O(n^3)$ // cubic



Complexity Examples

Big-O Complexity Chart



<http://bigocheatsheet.com/>

Complexity Examples

Array Sorting Algorithms

| Algorithm | Time Complexity | | | Space Complexity |
|----------------|-----------------|-------------------|-------------------|------------------|
| | Best | Average | Worst | Worst |
| Quicksort | $O(n \log(n))$ | $O(n \log(n))$ | $O(n^2)$ | $O(\log(n))$ |
| Mergesort | $O(n \log(n))$ | $O(n \log(n))$ | $O(n \log(n))$ | $O(n)$ |
| Timsort | $O(n)$ | $O(n \log(n))$ | $O(n \log(n))$ | $O(n)$ |
| Heapsort | $O(n \log(n))$ | $O(n \log(n))$ | $O(n \log(n))$ | $O(1)$ |
| Bubble Sort | $O(n)$ | $O(n^2)$ | $O(n^2)$ | $O(1)$ |
| Insertion Sort | $O(n)$ | $O(n^2)$ | $O(n^2)$ | $O(1)$ |
| Selection Sort | $O(n^2)$ | $O(n^2)$ | $O(n^2)$ | $O(1)$ |
| Tree Sort | $O(n \log(n))$ | $O(n \log(n))$ | $O(n^2)$ | $O(n)$ |
| Shell Sort | $O(n \log(n))$ | $O(n(\log(n))^2)$ | $O(n(\log(n))^2)$ | $O(1)$ |
| Bucket Sort | $O(n+k)$ | $O(n+k)$ | $O(n+k)$ | $O(n)$ |
| Radix Sort | $O(nk)$ | $O(nk)$ | $O(nk)$ | $O(n+k)$ |
| Counting Sort | $O(n+k)$ | $O(n+k)$ | $O(n+k)$ | $O(k)$ |
| Cubesort | $O(n)$ | $O(n \log(n))$ | $O(n \log(n))$ | $O(n)$ |

<http://bigocheatsheet.com/>

Why does it matter?

| Algorithm | 10 | 20 | 50 | 100 | 1,000 | 10,000 | 100,000 |
|----------------|------|----------|----------|----------|----------|----------|----------|
| $O(1)$ | <1 s | <1 s | <1 s | <1 s | <1 s | <1 s | <1 s |
| $O(\log(n))$ | <1 s | <1 s | <1 s | <1 s | <1 s | <1 s | <1 s |
| $O(n)$ | <1 s | <1 s | <1 s | <1 s | <1 s | <1 s | <1 s |
| $O(n \log(n))$ | <1 s | <1 s | <1 s | <1 s | <1 s | <1 s | <1 s |
| $O(n^2)$ | <1 s | <1 s | <1 s | <1 s | <1 s | 2 s | 3 m |
| $O(n^3)$ | <1 s | <1 s | <1 s | <1 s | 20 s | 6 h | 232 d |
| $O(2^n)$ | <1 s | <1 s | 260 d | ∞ | ∞ | ∞ | ∞ |
| $O(n!)$ | <1 s | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| $O(n^n)$ | 3 m | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |