

CS200 Fall 2014 WA2 due 10/7/14 by 9:30AM

Put both your name and your lab section at the top of your submission.

1. [15 points] Use mathematical induction to show that whenever  $n$  is a positive integer:

$$\frac{1}{1 * 3} + \frac{1}{3 * 5} + \dots + \frac{1}{(2n - 1)(2n + 1)} = \frac{n}{2n + 1}$$

- a) What is the basis step?
  - b) What is the inductive step?
2. [20 points] Give a big-O estimate for each of these functions. For the function  $g$  in your estimate  $f(x)$  is  $O(g)$ , use a simple function  $g$  of smallest order.
- a.  $6x^4 + 3^x + 12$
  - b.  $2 + 4x + 8x^2$
  - c.  $(x^3 + 2x)/(2x + 1)$
  - d.  $\log x + 27$
  - e.  $(2^x + x^2)(x^4 + 3^x)$
3. [12 points] In the manner of Examples 1 and 2 from section 3.2 of the Rosen text, show that  $4x^3 + 12x + 8$  is  $O(x^4)$ . Give an explanation and values of  $C$  and  $k$ .
4. [15 points] For the following equation:  $32x^4 + ((5x^2 + x)(13x^3 + 1))$
- a. State the closest  $g(x)$  for the equation.
  - b. Explain how the Big-O is computed given Theorems 1, 2 and 3 in Section 3.2 in Rosen.
5. [20 points] Assuming you use  $k=1$  as a witness, state whether the following functions are  $O(x^3)$ , and if so, state for what value of  $C$ .
- a.  $3^x$
  - b.  $10x + 42$
  - c.  $2 + 4x + 8x^2$
  - d.  $(\log x + 1)(2x + 3)$
  - e.  $2x + x!$
6. [18 points] Given the following algorithm, what is its Big-O? Justify your answer using the three theorems presented in the Complexity notes (and found in section 3.2 of Rosen). Assume the line “d” takes  $O(1)$  or  $O(c)$ .
- a. for (int x = 1; x <= n; ++x) {
  - b.     for (int y = 0; y < n; ++y) {
  - c.         for (int z = n; z > n-10; --z) {
  - d.             m = m \* aArray[x] \* bArray[y] \* cArray[z];
  - e.             }
  - f.         }
  - g.     }