

CS200 Fall 2014 HW 4 due 11/4/14 by **9:30AM**

Make sure you put your name and lab section on every sheet that you hand in.

Recurrence Relations and the Master Theorem

1. Given a recurrence relation, what are the first 5 elements of the sequence?
 - a. $a_n = 2a_{n-2}$, $a_0 = 3$, $a_1 = -1$
 - b. $a_n = a_{n-1} + a_{n-2} + n + 3$, $a_0 = 1$, $a_1 = 2$
2. For each of the recurrence relations in question 1, state whether it is linear and homogeneous. If not, state which condition is violated. If yes, what is the degree (k)?
3. Given a sequence, what is the recurrence relation?
 - a. $\{2, 5, 14, 41, 122\}$
 - b. $\{-1, 0, 1, 3, 13\}$
4. Given a recurrence relation, what is the closed form (Hint: Theorem 1 and 2)?
 - a. $a_n - a_{n-1} - 6a_{n-2} = 0$, $a_0 = 1$, $a_1 = 2$
 - b. $a_n = -6a_{n-1} - 9a_{n-2}$ for $n \geq 2$, $a_0 = 3$, $a_1 = -3$
5. Given an algorithm, how can it be characterized using the Master Theorem? For each of these give the values of a , b , $g(n)$ and *Big-O* as well as a prose description of how you determined them.
 - a. The end of the Trees lectures described a sort algorithm called Treesort (also found on page 624 in Prichard). Use the Master Theorem to analyze its complexity assuming that “copy” is the key operation.
 - b. Consider the problem of sorting a deck of playing cards by face value (ignoring suit) by placing them in a separate pile for each face value and then combining the piles. Use the Master Theorem to analyze the complexity assuming that the key operation is moving a card.
6. Let a_n denote the number of bit strings of length n in which contain three consecutive 1's.
 - a. Find the recurrence relation of a_n .
 - b. How many bit strings of length six contain three consecutive 1's?