08/2014
Colorado State University
CS200 - Howe

Grammar Worksheet

Warm Up Exercises:

This exercise pertains to the tortoise and the hare from Aesop's Fables.
The next three exercises refer to the grammar with:
Start symbol (S) = sentence
Set of terminals (T) = {the, sleepy, happy, tortoise, hare, passes, runs, quickly, slowly}
Set of nonterminals (N) = {noun, phrase, transitive verb phrase, intransitive verb phrase, article, adjective, noun, verb, adverb}
Productions (P) these are the rules associated with the grammar =

- sentence → noun phrase, transitive verb phrase, noun phrase
- sentence → noun phrase, intransitive, verb phrase
- noun phrase → article, adjective, noun
- noun phrase → article, noun
- transitive verb phrase → transitive verb
- intransitive verb phrase → intransitive verb, adverb
- intransitive verb phrase → intransitive verb

These are the valid words associated with the grammar:

- article → the
- adjective → sleepy
- adjective → happy
- noun → tortoise
- noun → hare
- transitive verb → passes
- intransitive verb → runs
- adverb → quickly
- adverb → slowly

(1) Use the set of productions to show that these are valid sentences:
Example) The happy hare runs

sentence				
noun phrase			intransitive verb phrase	
article	adjective	noun	intransitive verb phrase	
article	adjective	noun	intransitive verb	
the	happy	hare	runs	

a) The sleepy tortoise runs quickly

sentence				
noun phrase			intransitive verb phrase	
article	adjective	noun	intransitive verb phrase	
article	adjective	noun	intransitive verb	adverb
The	sleepy	tortoise	runs	quickly

b) The tortoise passes the hare

sentence
→ noun phrase transitive verb phrase noun phrase

→	article	noun	transitive verb phrase	noun phrase	
→	article	noun	transitive verb	noun phrase	
→	article	noun	transitive verb	article	noun
	the	tortoise	passes	the	hare

c) The sleepy hare passes the happy tortoise

	sentence						
→	noun phrase			transitive verb phrase	noun phrase		
→	article	adjective	noun	transitive verb phrase	noun phrase		
→	article	adjective	noun	transitive verb	noun phrase		
→	article	adjective	noun	transitive verb	article	adjective	noun
	the	sleepy	hare	passes	the	happy	tortoise

(2) Find three additional valid sentences:

Several possible answers. Some of these answers include:
The sleepy hare runs quickly
The hare passes the tortoise
The happy hare runs slowly
The happy tortoise passes the hare
The hare passes the happy hare

(3) Show that "the hare runs the sleepy tortoise" is not a valid sentence:

The only way to get a noun, such as tortoise, at the end is to have a noun phrase at the end, which can be achieved only via the production:

sentence		
noun phrase	transitive verb phrase	noun phrase

However, the only valid transitive verb we have is "passes" (runs is an intransitive verb). Thereby this sentence is not valid within the grammar defined.

In Recitation Exercises:

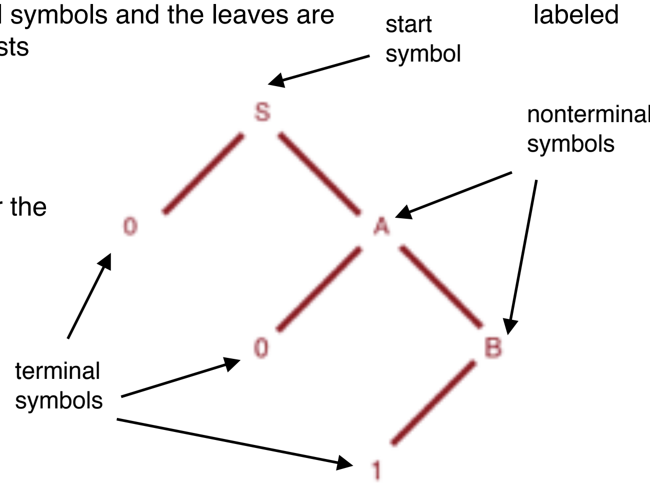
(4) Let G = (V, T, S, P) be the grammar with V = {0, 1, A, B, S}, T = {0,1}, and set of productions P consisting of $S \rightarrow 0A$, $S \rightarrow 1A$, $A \rightarrow 0B$, $B \rightarrow 1A$, $B \rightarrow 1$

a) What is the language generated by G?

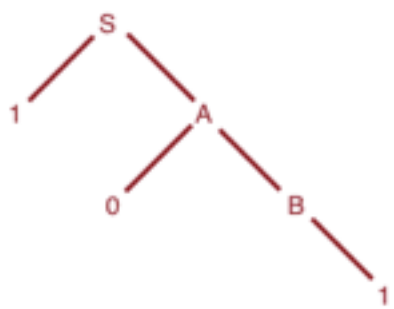
All strings consisting of a 0 or a 1 followed by one or more repetitions of 01

b) Derivation Trees: a derivation tree has a specific format; the root represents the start symbol, internal nodes are labeled with nonterminal symbols and the leaves are labeled with terminal symbols. For each sentence there exists at least one derivation tree but you always need a single grammatically correct sentence to create a derivation tree.

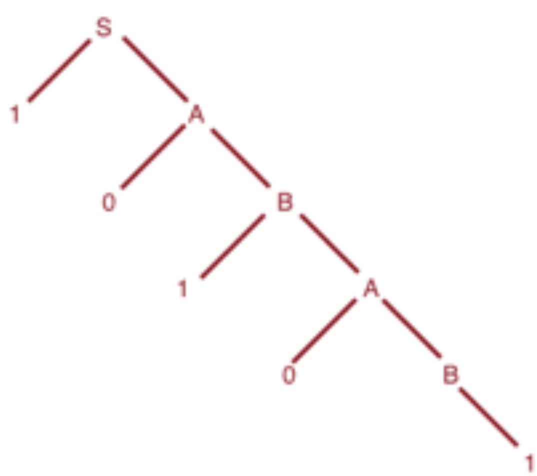
The example to the right shows a derivation tree for the sentence 001 that uses the grammar in question 4:



Draw the derivation tree associated with the sentence 101:



Draw the derivation tree associated with the sentence 10101:



(5) Let V = {S,A,B,a,b} and T = {a,b}. Find the language generated by the grammar (V,T,S,P) when the set P of productions consists of:

a) S → AB, S → aA, A → a, B → ba

This time there are only two possible strings: {aa, aba}

b) S → AA, S → B, A → aaA, A → aa, B → bB, B → b

If S → AA is applied then the string results must be N number of a's where N is an even number greater or equal to 4 since each A because a positive even number of As

If S → B is applied then the result is a string of one or more b's

Therefore the language is $\{ A^{2n} \mid n >= 2 \} \cup \{ b^n \mid N >= 1 \}$

(6) Find the grammar for the language with the set of all bit strings containing an even number of 0s and no 1s:

<S> = 00<S> | λ

alternatively:
<S> = 00<A>
<A> = 00<A> | λ

a) the set consisting of the strings 0, 11, and 010

<S> = 0 | 11 | 010

b) the set of strings of three 0s followed by two or more 0s

<S> = 0000<A>
<A> = 0<A> | 0

c) the set of strings that contain any number of 0s and exactly one 1

<S> = 0<S> | 1<A>
<A> = 0<A> | λ

d) The set of odd-length strings whose first, middle, and last characters are all the same, over the alphabet {0,1} (Some examples include: 000, 01000, 10111, 1011)

<S> = 0<A>0 | 1<B>1 | 0 | 1
<A> = 0<A>0 | 0<A>1 | 1<A>0 | 1<A>1 | 0
<B> = 0<B>0 | 0<B>1 | 1<B>0 | 1<B>1 | 1

(7) A palindrome is a string that reads the same backward as it does forward, that is, a string w, where w = w^R, where w^R is the reversal of the string w. Find that grammar that generates the set of odd length palindromes over the alphabet {a,b}:

NOTE: The major difference between odd palindromes and palindromes of any length is that you cannot have an empty string

<pal> = <ch> | a <pal> a | b <pal> b
<ch> = a | b