Fall 2014

More About Induction

Recitation 4

Example - Induction in excruciating detail

I will admit to getting this example from Wikipedia [1], but it is a good place to start. The following is the formula, called p(n), for the summation of natural numbers:

$$0 + 1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$$

Basis step: Show that it holds for n = 0

$$0 = \frac{0 * (0+1)}{2}$$
$$0 = \frac{0}{2} = 0$$

Start with only 0 on the left side, and $n = 0$ on the right side of the previous equation. Do the arithmetic and if the left side equals the right side, you have proved the basis. Or show that $p(0)$ holds, depending on your terminology.

Inductive Step: We now need to show this if $p(k)$ holds, then $p(k+1)$ also holds. See later note about n and k. *Assume that p(k) holds, for some unspecified value of k.* What we want to show is that $p(k+1)$ holds:

$$(0 + 1 + 2 + ... + k) + (k+1) = \frac{(k+1)((k+1)+1)}{2}$$

Notice what was done here. The left side started out the same as the left side of the very first equation, and then had a $'(k+1)'$ added to it. The right side started out the same as the right side of the very first equation. Then, every k was robotically replaced with a $'(k+1)'$.

We already have a formula for $p(k)$, from the original equation. We can substitute that for the $'(0 + 1 + 2 + ... + k)'$ term above:

$$\frac{k(k+1)}{2} + (k+1) = \frac{(k+1)((k+1)+1)}{2}$$

What we must now do is make the left side match the right side.

$$\frac{k(k+1) + 2(k+1)}{2} = \frac{(k+1)((k+1)+1)}{2}$$

$$\frac{k^2+3k+2}{2} = \frac{(k+1)((k+1)+1)}{2}$$
$$\frac{(k+1)(k+2)}{2} = \frac{(k+1)((k+1)+1)}{2}$$
$$\frac{(k+1)((k+1)+1)}{2} = \frac{(k+1)((k+1)+1)}{2}$$

Left now equals right, we are done. We have shown that $p(k+1)$ holds by showing that the left and right sides of the original equation are still equivalent in the $(k+1)$ case just like they were for k.

About k and n, the notation changes depending on which part of the problem is being done. n is usually used for describing the original equation. k is used during the algebra for the inductive step, but it is the same variable renamed. I think this is to create visual distinction between the original equation and the later algebra work. For our purposes n == k.

Exercise for today:
The work sheet with more induction practice. This is also your attendance sheet, show it to me when you are done or at the end of the recitation and I will record attendance. You may keep the worksheet if your want to.

Optional extra problem:
If you are done with the worksheet with time to spare, and want to do some programming, try this problem. It's not worth any points, but it's good practice.

- Look up the Lucas Numbers on Wikipedia: `http://en.wikipedia.org/wiki/Lucas_number`
- This is a series of numbers similar to the Fibonacci series.
- Make a new eclipse project, and write a recursive function which computes the n^{th} Lucas Number.
- This description has been left intentionally vague