
Recitation 6 Worksheet 1

Complexity

1. What is big-O for each of the following?

- $f(x) = 17 + 3x + 21x^2$
- $g(x) = 19x + x(2x + 2)$
- $h(x) = x^{\frac{n}{2}} + x^{\frac{n}{3}} + x^{\frac{n}{4}}$ for some $n \geq 1$
- $j(x) = x^2 \log(x)$

2. Show that $g(x) = 19x + x(2x + 2)$ is not $O(x)$

3. Using $k = 1$, show that $h(x) = x^{\frac{n}{2}} + x^{\frac{n}{3}} + x^{\frac{n}{4}}$ for some $n \geq 1$ is $O(x^n)$ and state a witness C that can be used with $k = 1$ to demonstrate this fact.

4. Given an integer array $a[]$ of size n , What are the worst-case, best-case, and average-case complexity of the following?

```
for (int i=0; i<n; i++) {
    for (int j=0; j<n; j++) {
        if (i!=j && a[i]==a[j]) return a[i];
    }
}
```

5. Find a recurrence relation for the number of strings of digits that contain exactly one zero or one digit.

6. Use the master theorem to compute the complexity of the following:

```
int foo(int[] a) {
    if (a.length == 1) return a[0];
    if (a.length == 2) return a[0] + a[1];
    int size = a.length/3;
    int extra = a.length%3;
    int[] a0 = new int[size];
    int[] a1 = new int[size];
    int[] a2 = new int[size+extra];
    for (int i=0; i<size; i++) {
        a0[i] = a[i];
        a1[i] = a[i+size];
        a2[i] = a[i+2*size];
    }
    for (int i=0; i<extra; i++) {
        a2[i+size] = a[i+3*size];
    }
    return foo(a0) + foo(a1) + foo(a2);
}
```