Graphs
Recitation 13

Basic Terminology:

Graphs represent relationships among data items. They are comprised of a set of V, **vertices (nodes)**, and a set of **E, edges**. Each edge represents a connection between two vertices.

Use the class notes or Rosen/Prichard define the following terms:
Adjacent:

Incident:

Path:

Simple path:

Cycle:

Connected:

Strongly connected:

Connected components:

Weakly connected:

Complete:

Degree:

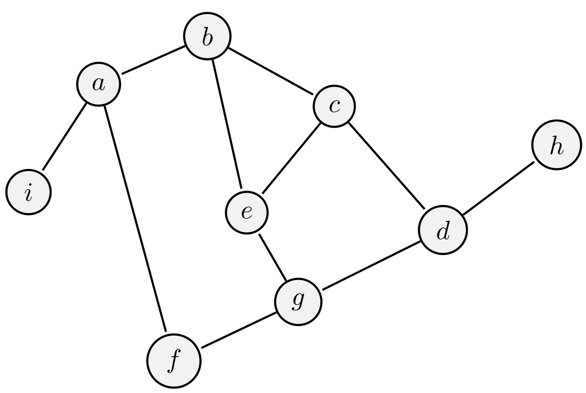
Indegree:

Outdegree:

Representations of Graphs:

Adjacency matrix: Typically represented as an array of arrays. The two examples below correspond to an unweighted undirected graph and an unweighted directed graph. 1 corresponds to an edge being present and a 0 corresponds to no edge being present. Notice how in a directed graph if $a \rightarrow b$ then $matrix[a][b] = 1$ and $matrix[b][a] = 0$.
Adjacency list: Typically represented as a List of Lists. Each node has a corresponding list of all the nodes it is connected with. Notice how in the directed graph is $a \rightarrow b$ then a contains b in its list but b does not contain a.

Representation of an undirected graph



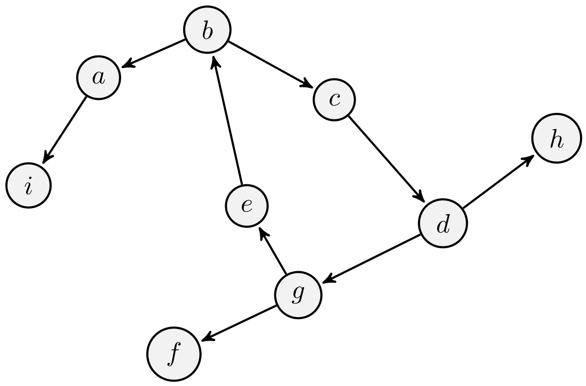
(a) undirected graph

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
a	0	1	0	0	0	1	0	0	1
b	1	0	1	0	1	0	0	0	0
c	0	1	0	1	1	0	0	0	0
d	0	0	1	0	0	0	1	1	0
e	0	1	1	0	0	0	1	0	0
f	1	0	0	0	0	0	1	0	0
g	0	0	0	1	1	1	0	0	0
h	0	0	0	1	0	0	0	0	0
i	1	0	0	0	0	0	0	0	0

(b) adjacency matrix representation

a	$\rightarrow$ b $\rightarrow$ f $\rightarrow$ i
b	$\rightarrow$ a $\rightarrow$ c $\rightarrow$ e
c	$\rightarrow$ b $\rightarrow$ d $\rightarrow$ e
d	$\rightarrow$ c $\rightarrow$ g $\rightarrow$ h
e	$\rightarrow$ b $\rightarrow$ c $\rightarrow$ g
f	$\rightarrow$ a $\rightarrow$ g
g	$\rightarrow$ d $\rightarrow$ e $\rightarrow$ f
h	$\rightarrow$ d
i	$\rightarrow$ a

(c) adjacency list representation



(a) directed graph

	a	b	c	d	e	f	g	h	i
a	0	0	0	0	0	0	0	0	1
b	1	0	1	0	0	0	0	0	0
c	0	0	0	1	0	0	0	0	0
d	0	0	0	0	0	0	0	1	0
e	0	1	0	0	0	0	0	0	0
f	0	0	0	0	0	0	0	0	0
g	0	0	0	0	1	1	0	0	0
h	0	0	0	0	0	0	0	0	0
i	0	0	0	0	0	0	0	0	0

(b) adjacency matrix representation

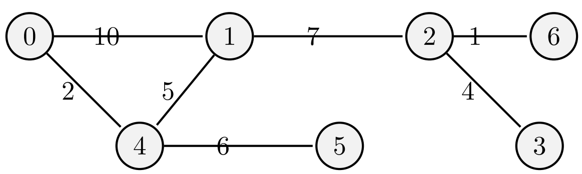
Representation of an directed graph

a	→ i
b	→ a → c
c	→ d
d	→ h
e	→ b
f	
g	→ d → e
h	
i	

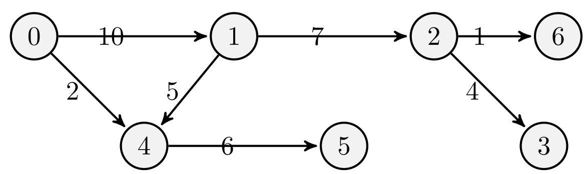
(c) adjacency list representation

Additional Problems:

1. Now consider the previous representations would change if the graph is weighted.
   Give the adjacency list and the adjacency matrix for the following graphs:



(a) graph 1



(b) graph 2

`Adjacency Matrix for Graph 1:`

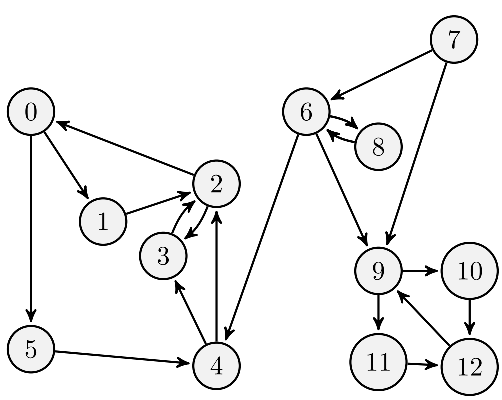
`Adjacency List for Graph 1:`

`Adjacency Matrix for Graph 2:` `Adjacency List for Graph 2:`

2. Answer the following questions pertaining to graph 1 and 2:
   (a) Both graphs: is the adjacency matrix be symmetrical? When is an adjacency matrix symmetrical?
   (b) Both graphs: what does the sum of each row of the adjacency matrix represent?
   (c) Graph 1: List the degree of each of the nodes
   (d) Graph 2: List the indegree and outdegree of each of the nodes

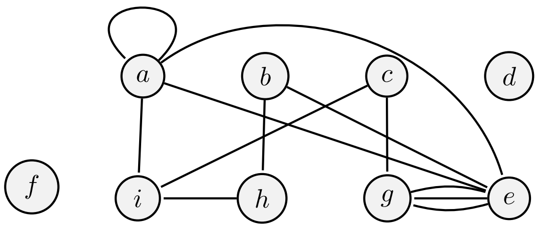
3. Would an adjacency matrix or an adjacency list be a better implementation for the following graph operations? Please explain your answer:
   - Determine whether there is an edge from vertex i to vertex j
   - Find all the vertices adjacent to a given vertex i

4. Find all the strongly connected components (SCC's) in this digraph:



(a) graph 3

5. Identify:
   (a) the number of vertices:
   (b) the number of edges:
   (c) the degree of each vertex:
   (d) isolated vertices
   (e) pendent vertices

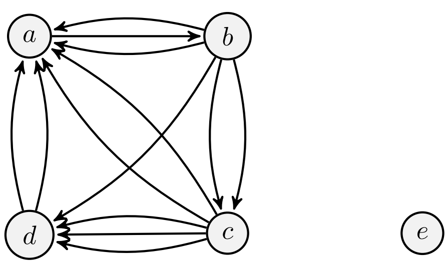


(a) graph 4

6. Find the sum of the degrees of vertices and verify that it equals twice the sumer of edges in the graph.

7. Can a simple graph exist with 15 vertices each of degree five?

8. Identify:
   (a) the number of vertices
   (b) the number of edges
   (c) the in-degree and out-degree of each vertex



(a) graph 5

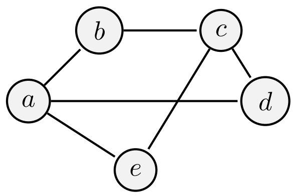
9. next to each vertex show write the in-degree and the out degree. Sum these up and show that they are both equal to the number of edges in the graph.

10. Draw the following graphs:
    (a) C_7

(b) $K_{4,4}$

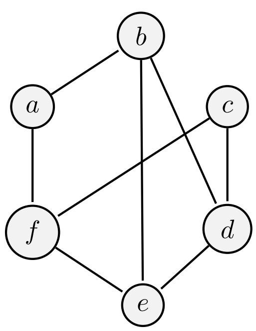
11. What does bipartite mean:

    Determine whether the following graph is bipartite



(a) graph 5

12. For which values of n is this graph bipartite?
    (a) K_n
    (b) C_n
    (c) W_n
    (d) Q_n



(a) graph 7

13. How many vertices and how many edges do these graphs have?
    (a) K_n
    (b) C_n
    (c) W_n
    (d) $K_{m,n}$
    (e) Q_n