Divide and Conquer Algorithms: Advanced Sorting

Prichard Ch. 10.2: Advanced Sorting Algorithms

(revisit) Properties of Growth-rate functions(1/3)

1. You can ignore low-order terms in an algorithm's growth-rate function.

- $O(n^3+4n^2+3n)$ it is also $O(n^3)$

(revisit) Properties of Growth-rate functions(2/3)

2. You can ignore a multiplicative constant in the high-order term of an algorithm's growth-rate function

- $O(5n^3)$, it is also $O(n^3)$

(revisit) Properties of Growth-rate functions (3/3)

3. You can combine growth-rate functions

- $O(n^2) + O(n)$, it is also $O(n^2+n)$
- Which you write as $O(n^2)$

Examples

Find a growth function that has the ***best*** estimation of $O(x^2)$.

$A.\ f(x) = 17x + 11$

$B.\ f(x) = x^2 + 1000$

$C.\ f(x) = xlogx$

$D.\ f(x) = x^4/2$

$E.\ f(x) = 2^x$

Demonstrating Efficiency

- Computational complexity of the algorithm
  - Time complexity
  - Space complexity
    - Analysis of the computer memory required
    - Data structures used to implement the algorithm

Best, Average, and Worst Cases

- **Worst case**
  - Just how bad can it get:
    - The maximal number of steps
- **Average case**
  - Amount of time expected “usually”
- **Best case**
  - The smallest number of steps

Sequential Search

10	**9**	**3**	**8**	**5**	**6**	**7**	**4**	**1**	**45**	**90**	**22**	**2**	**0**

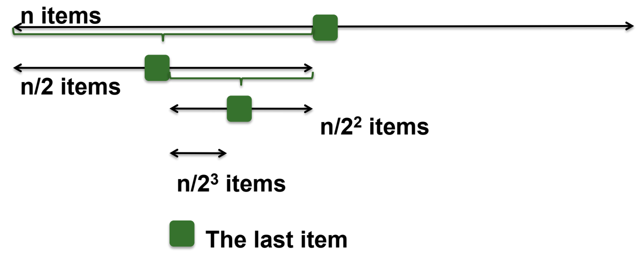
- Array of n items
  - From the first one until either you find the item or reach the end of the array.
  - Best case: $O(1)$
  - Worst case: $O(n)$ (n times of comparison)
  - Average case: $O(n)$ ($n/2$ comparison)

Binary Search (1/4)

10	11	13	15	16	20	22	39	40	45	90	92	93	94

- Searches a ***sorted array*** for a particular item by repeatedly dividing the array in half.
- Determines which half the item must be in and discards other half.
- Suppose that $n = 2^k$ for some k. (n=1,2,4,8,16,…)
  1. Inspect the middle item of size n
  2. Inspect the middle item of size $n/2$
  3. Inspect the middle item of size $n/2^2$
  4. .
  5. .
  6. .

Binary Search (2/4)



If we have n = 2^k, in worst case, it will repeat this k times

Clicker Q

- What is the worst case for binary search?
  A. Item is at the end of the array
  B. Item is at the beginning of the array
  C. Item is not in the array
  D. The array is not sorted

Binary Search (3/4)

- Dividing array in ***half*** k times.
- Worst case
  - Algorithm performs k divisions and k comparisons.
  - Since $n = 2^k$, $k = \log_2 n$
  - $O(\log_2 n)$

Binary Search (4/4)

- What if n is not a power of 2?
- We can find the smallest k such that,

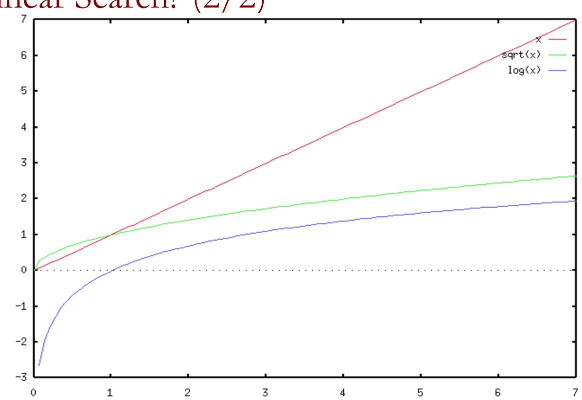
$2^{k-1} < n < 2^k$
$k - 1 < \log_2 n < k$
$k < 1 + \log_2 n < k+1$
$k = 1 + \log_2 n$ rounded down

Therefore, the algorithm is still $O(\log_2 n)$.

Is Binary Search is more Efficient than Linear Search? (1/2)

- For large number, $O(\log_2 n)$ requires significantly less time than $O(n)$
- For small numbers such as $n < 25$, does not show big difference.

Is Binary Search is more Efficient than Linear Search? (2/2)



Sorting Algorithm

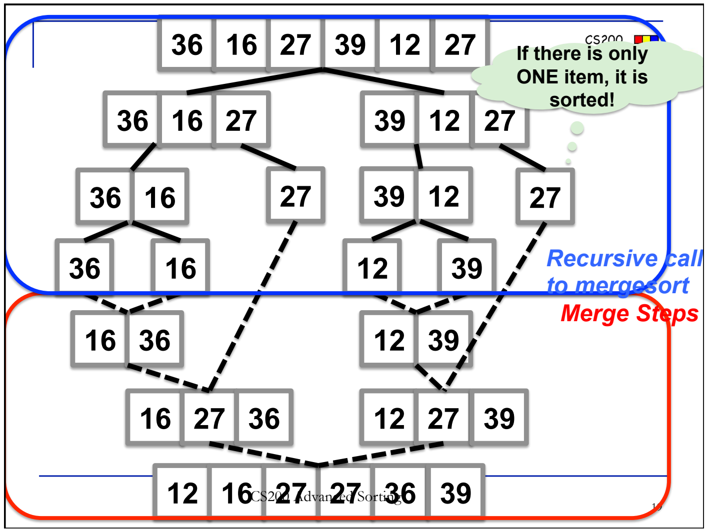
- Organize a collection of data into either ***ascending*** or ***descending*** order.
- *Internal sort*
  - Collection of data fits entirely in the computer's main memory
- *External sort*
  - Collection of data will not fit in the computer's main memory all at once.
- We will only discuss **internal sort**.

Aside: Sorting Redux from 161

- Simple Sorts: Bubble, Insertion, Selection
- Doubly nested loop
- Outer loop puts one element in its place
- It takes i steps to put element i in place
  - n-1 + n-2 + n-3 + … + 3 + 2 + 1
  - $O(n^2)$ complexity
  - In place

Mergesort

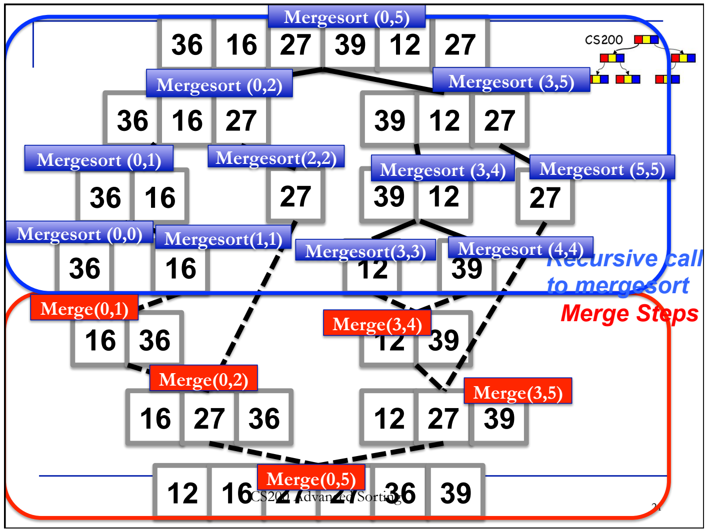
- ***Recursive sorting algorithm***
- Gives the **same** performance
- ***Divide-and-conquer***
  - Step 1. Divide the array into halves
  - Step 2. Sort each half
  - Step 3. Merge the sorted halves into one sorted array



MergeSort code

```
public static void mergesort(Comparable[] theArray, int first, int
   last){
   // Sorts the items in an array into ascending order.
   // Precondition: theArray[first..last] is an array.
   // Postcondition: theArray[first..last] is sorted.
   if (first < last) {
         int mid = (first + last) / 2;  // midpoint of the array
         mergesort(theArray, first, mid);
         mergesort(theArray, mid + 1, last);
         merge(theArray, first, mid, last);
   }// if first >= last, there is nothing to do
}
```

why does it work?



Clicker Q

- How many times was MergeSort called?
  A. 1
  B. 6
  C. 10
  D. 20

Merge code I

```
private static void merge (Comparable[] theArray, Comparable[]
   tempArray, int first, int mid, int last({

  int first1 = first;
  int last1 = mid;
  int first2 = mid+1;
  int last2 = last;
  int index = first1;   // incrementally creates sorted array

  while ((first1 <= last1) && (first2 <= last2)){
    if( theArray[first1].compareTo(theArray[first2])<0) {
      tempArray[index] = theArray[first1];
      first1++;
    }
    else{
      tempArray[index] = theArray[first2];
      first2++;
    }
    index++;
  }
```


Merge code II

```
  // finish off the two subarrays, if necessary
  while (first1 <= last1){
    tempArray[index] = theArray[first];
    first1++;
    index++;  }
  while(first2 <= last2)
    tempArray[index] = theArray[first2];
    first2++;
    index++;  }
  for (index = first; index <= last: ++index){
    theArray[index ] = tempArray[index];
  }
} //end merge
```

Mergesort Complexity

- Analysis
  - Merging:
    - for total of n items in the two array segments, at most n -1 comparisons are required.
    - n moves from original array to the temporary array.
    - n moves from temporary array to the original array.
    - Each merge step requires $3n - 1$ major operations

Mergesort: More complexity

- Each call to `mergesort` recursively calls itself **twice**.
- Each call to `mergesort` **divides** the array into two.
  - First time: divide the array into 2 pieces
  - Second time: divide the array into 4 pieces
  - Third time: divide the array into 8 pieces

Mergesort Levels

- If n is a power of 2 (i.e. $n = 2^k$), then the recursion goes $k = \log_2 n$ levels deep.
- If n is not a power of 2, there are $1 + \log_2 n$ (rounded down) levels of recursive calls to `mergesort`.

Mergesort Operations

- At level 0, the original call to `mergesort` calls merge once. (requires $3n - 1$ operations)
- At level 1, two calls to `mergesort` and each of them will call `merge`.
  - Total $2 * (3 * (n/2) - 1)$ operations required
- At level m, 2^m calls to `merge` occur.
  - Each of them will call merge with $n/2^m$ items and each of them requires $3(n/2^m)$-1 operations. Together, $3n$-2^m operations are required.
- Because there are $log_2 n$ or 1+$log_2 n$ levels, total $O(n*log_2 n)$

Mergesort Computational Cost

- Since there are either $\log_2 n$ or $1+\log_2 n$ levels, `mergesort` is $O(n*\log_2 n)$ in both the **worst** and **average** cases.
- **Significantly faster** than $O(n^2)$

Clicker Q

- Is MergeSort *O(nlogn)* in the best case?

A. Yes

B. No

Stable Sorting Algorithms

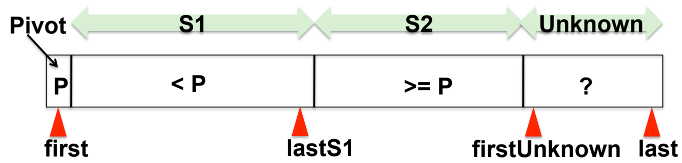
- Suppose we are sorting a database of users according to their name. Users can have identical names.
- *A* stable sorting algorithm maintains the relative order of records with equal keys (i.e., sort key values). Stability: whenever there are two records *R* and *S* with the same key and *R* appears before *S* in the original list, *R* will appear before *S* in the sorted list.
- Is mergeSort stable?

Quicksort

1. Select a **pivot** item.
2. Subdivide array into 3 parts
   - **Pivot in its "sorted" position**
   - Subarray with **elements < pivot**
   - Subarray with **elements >= pivot**
3. **Recursively** apply to each sub-array

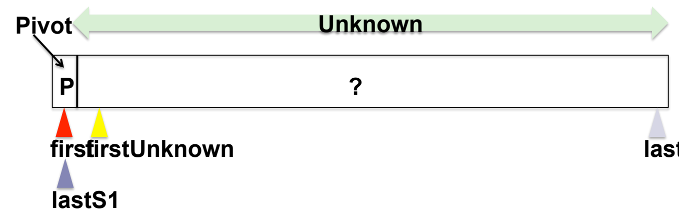
Invariant for partition

CS200



Initial state of the array

CS200



Lecture 12: 10/2/14

CS200

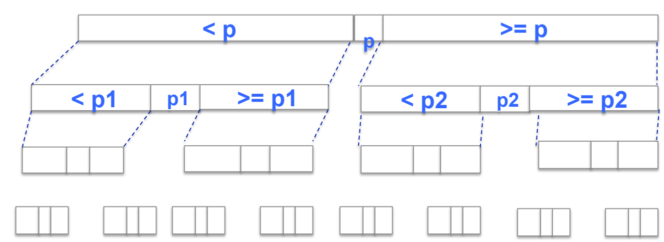
- Grammars (Prichard Ch. 6.2, Rosen Ch. 13.1)
- Stacks (Prichard Ch. 7)
- Recursion (Prichard Ch. 6.1 & 6.3)
- Queues (Prichard Ch. 8)
- Complexity (Rosen Ch. 3.2, 3.3, Prichard 10.1)
- Advanced Sorting (Prichard 10.2)

- PA2 LATE PERIOD.
- WA2 due 10/7 @ 9:30
- Midterm 1 on 10/9

Today's Trivial Participation Quiz:
*Which password was *not* on 2014 25 Most Common Passwords list?*
a. letmein
b. trustno1
c. monkey
d. guessit

Quicksort Key Idea: Pivot

CS200



Clikcer Q

- An invariant for the QuickSort code is:

A. After the first pass, the P< partition is fully sorted.
B. After the first pass, the P>= partition is fully sorted.
C. After each pass, the pivot is in the correct position.
D. It has no invariant.

QuickSort Code

```
public static void quickSort(Comparable[] theArray, int first,
                                   int last) {
    int pivotIndex;
    if (first < last) {
    // create the partition: S1, Pivot, S2
      pivotIndex = partition(theArray, first, last);
    // sort regions S1 and S2
      quickSort(theArray, first, pivotIndex-1);
      quickSort(theArray, pivotIndex+1, last);
    } // end if
  } // end quickSort
```


Partition Overview

1. Choose and position pivot
2. Take a pass over the current part of the array
    1. If item < pivot, move to S1 by incrementing S1 last position and swapping item into beginning of S2
    2. If item >= pivot, leave where it is
3. Place pivot in between S1 and S2

Partition Code: the Pivot

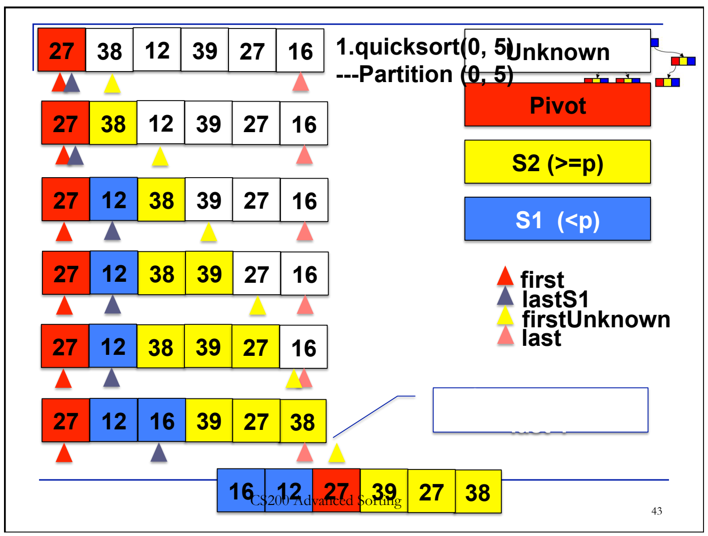
```
private static int partition(Comparable[] theArray, int first, int last) {
     Comparable tempItem;
   // place pivot in theArray[first]
   // by default, it is what is in first position
     choosePivot(theArray, first, last);
     Comparable pivot = theArray[first]; // reference pivot
   // initially, everything but pivot is in unknown
     int lastS1 = first; // index of last item in S1
```

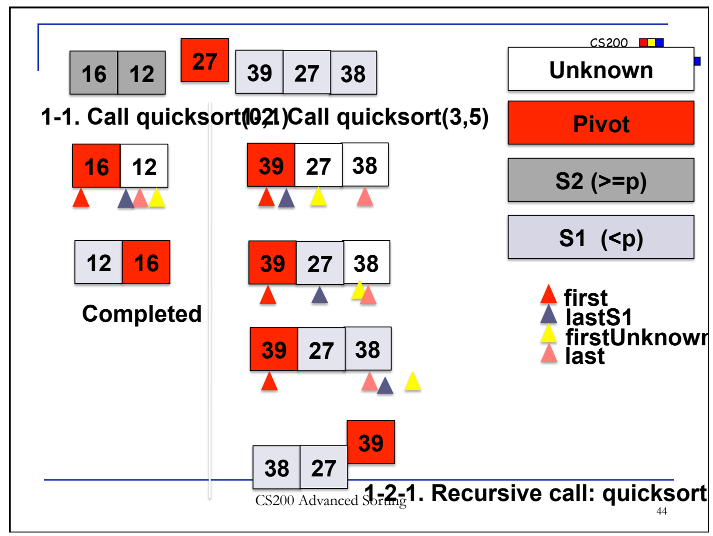
Partition Code: Segmenting

```
// move one item at a time until unknown region is empty
    for (int firstUnknown = first + 1; firstUnknown <= last; ++firstUnknown)
    {// move item from unknown to proper region
        if (theArray[firstUnknown].compareTo(pivot) < 0) {
            // item from unknown belongs in S1
            ++lastS1;  // figure out where it goes
            tempItem = theArray[firstUnknown]; // swap it with first unknown
            theArray[firstUnknown] = theArray[lastS1];
            theArray[lastS1] = tempItem;
            } // end if
        // else item from unknown belongs in S2 – which is where it is!
    } // end for
```


Partition Code: Replace Pivot

```
// place pivot in proper position and mark its location
    tempItem = theArray[first];
    theArray[first] = theArray[lastS1];
    theArray[lastS1] = tempItem;
    return lastS1;
  } // end partition
```

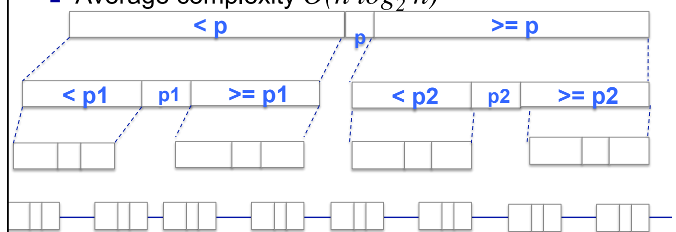


Quicksort Visualizations

- http://en.wikipedia.org/wiki/Quicksort
- http://www.sorting-algorithms.com
- Hungarian Dancers via YouTube

Average Case

- Each **level** involves,
  - Maximum $(n-1)$ comparisons.
  - Maximum $(n-1)$ swaps. $(3(n-1)$ data movements)
  - $log_2 n$ levels are required.
- Average complexity $O(n \log_2 n)$



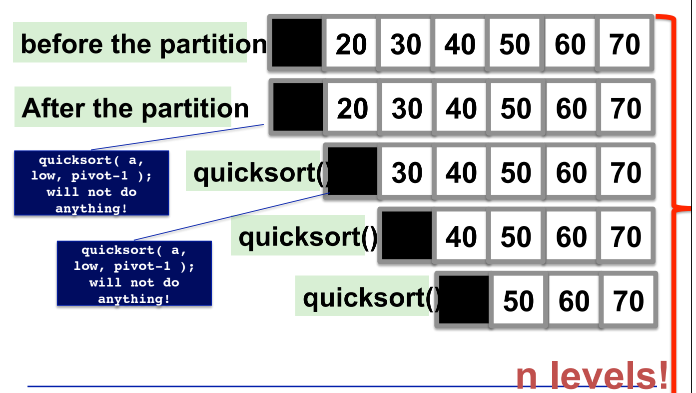
Clicker Q

- Is QuickSort like MergeSort in that it is always O(nlogn) complexity?

A. Yes

B. No

Worst Case!



Worst case analysis

- This case involves $(n\text{-}1)+(n\text{-}2)+(n\text{-}3)+\ldots+1+0 = n(n\text{-}1)/2$ comparisons
- `Quicksort` is $O(n^2)$ for the **worst-case**.

Selecting pivot

- Strategies for Selecting **pivot**
  - First value : worst case if the array is **sorted.**
- Middle value or Median value
  - Better for the sorted data

quickSort – Algorithm Complexity

- Depth of call tree?
    - O(log n) *split roughly in half,* best case
    - O(n) worst case
  - Work done at each depth
    - O(n)
  - Total Work
    - O(n log n) best case
    - $O(n^2)$ worst case

Clicker Q

- Why would someone pick QuickSort over MergeSort?

A. Less space
B. Better worst case complexity
C. Better average complexity
D. Easier to code

How fast can we sort?

- Observation: all the sorting algorithms so far are *comparison sorts*
  - A comparison sort must do at least O(n) comparisons (*why?*)
  - We have an algorithm that works in O(n log n)
  - What about the gap between O(n) and O(n log n)
- **Theorem:** all comparison sorts are Ω(n log n)
- MergeSort is therefore an "optimal" algorithm

Radix Sort (by MSD)

1. Take the most significant digit (MSD) of each number.
2. Sort the numbers based on that digit, grouping elements with the same digit into one bucket.
3. Recursively sort each bucket, starting with the next digit to the right.
4. Concatenate the buckets together in order.

80 24 62 40 68 20 26

	24, 20, 26	40	62, 68	80

20		24	26		40						62			68	80				

Radix Sort

- To avoid using extra space: Radix sort by Least Significant Digit

```
RadixSort(A, d)
     // d - number of digits
     for i=1 to d
          sort(A) on the ith least
               significant digit
```

Assumption: `sort(A)` is a stable sort

Show Example.

What to do if not all numbers have the same # of digits?

Radix sort

- Analysis
  - n moves each time it forms groups
  - n moves to combine them again into one group.
  - Total $2n*d$ (for the strings of d characters)
  - Radix sort is $O(n)$ for $d << n$

Radix Sort

- Radix sort is
  - Fast
  - Asymptotically fast (i.e., O(*n*))
  - Simple to code
  - A good choice
- Can we use it for strings?
- So why not use it for every application?