[Problem 1] (3+4) pts

Given the following data: 10, 15, 5, 18, 14, 6, 20, 9
a. What binary search tree is created by inserting the data in the order given?

b. Given a search key of 15, trace the algorithm that delete the item in the binary search tree that you created in Part a. List the nodes in the order in which the search visits them.

[Problem 2] (2+2+2) pts

[Diagram of a binary tree]

a. What is the preorder traversal of this binary tree?
b. What is the inorder traversal of this binary tree?
c. What is the postorder traversal of this binary tree?

[Problem 3] 5 pts

Prove by mathematical induction that a binary tree with n nodes has exactly n + 1 empty subtrees.
[Problem 4] (5+5) pts
Consider two algorithms for traversing a binary tree. Both are nonrecursive algorithms that use an extra data structure for bookkeeping. Both algorithms have the following basic form:

```
Put the root of the tree in the data structure
while (the data structure is not empty)
  Remove a node from the data structure and call it n
  Visit n
  if (n has a left child)
    Put the child in the data structure
  }
  if (n has a right child)
    put the child in the data structure
```}

The difference between the two algorithms is the method for choosing a node n to remove from the data structure.

*Algorithm 1*: Remove the newest (most recently added) node from the data structure
*Algorithm 2*: Remove the oldest (earliest added) node from the data structure

a. In what order would each algorithm visit the nodes of the tree in the tree below?

b. For each algorithm, describe an appropriate data structure for doing the bookkeeping.