Worksheet for Recursive relations and Master Theorem

1. Find a closed form of f(n) when $n= 2^k$, where f satisfies the recurrence relation $f(n) = f(n/2) +1$ with $f(1) = 1$

2. Find the Big O estimation when $n= 2^k$, where f satisfies the recurrence relation $f(n) = f(n/2) +1$ with $f(1) = 1$ using the Master Theorem

3. Find the complexity of merge sort using the Master Theorem.
   (1) Draw the recurrence tree (start with an array with 8 items)
   (2) Build a recurrence relation for merge sort
   (3) Find the complexity using the Master Theorem.

4. Find *f(n)* when $n = 3^k$, where *f* satisfies the recurrence relation, $f(n) = 2\,f(n/3) + 4$ with $f(1) = 1$.

   (1) Calculate *f(3), f(9), and f(27)* from the recurrence relation.

   (2) Find a closed form of *f(n).*

5 Assume that the population of the world in 2002 was 6.2 billion and is growing at the rate of 1.3% a year

   (1) Set up a recurrence relation for the population of the world n years after 2002.
   (2) Find an explicit formula for the population of the world n years after 2002

FAQ
How can we find the witnesses for the Big-O notation?
Example] Show that $f(x) = x^2+2x+1$ is $O(x^2)$
Show that $(x^2+4x+17)$ is $O(x^3)$.