

## CS200 Spring 2015 Recitation 9: Recurrences

**name:**

**id:**

### 1. Using the Master Theorem:

Let  $f$  be an increasing function that satisfies

$$f(n) = a \cdot f(n/b) + c \cdot n^d$$

whenever  $n = b^k$ , where  $k$  is a positive integer,  $a \geq 1$ ,  $b$  is an integer  $> 1$ , and  $c$  and  $d$  are real numbers with  $c$  positive and  $d$  nonnegative. Then

$$f(n) = \begin{cases} O(n^d) & \text{if } a < b^d \\ O(n^d \log n) & \text{if } a = b^d \\ O(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$

What are the big-O bounds recurrence relations?

a)  $f(n) = 3f(n/3) + 5n$

b)  $f(n) = 16f(n/4) + 8$

c)  $f(n) = f(n/2) + n^3$

d)  $f(n) = 2f(n/2) + 1$

### 2. Find a solution to the following recurrence relation, using repeated substitution:

$$f(1) = 1$$

$$f(n) = 4f(n-1) \quad \text{for } n > 1$$

3. Given the following method:

```
public int recSrch (int[] A, int k){
    return recSrch(A,k,0,A.length-1);
}
private int recSrch(int[]A, int k, int lo, int hi){
    if(lo>hi) return -1;
    else{
        int mid = (lo+hi)/2;
        if(k==A[mid]) return mid;
        int i1 = recSrch(A,k,lo,mid-1);
        int i2 = recSrch(A,k,mid+1,hi);
        return Math.max(i1,i2);
    }
}
```

a) Derive a recurrence  $rS(n)$  relation for  $\text{recSrch}(A, k)$ , where  $n$  is the length of  $A$ .

$$rS(n) = 1 \qquad \text{for } n = 1$$

$$rS(n) = \qquad \text{for } n > 1$$

b) Use the Master Theorem to solve the recurrence.