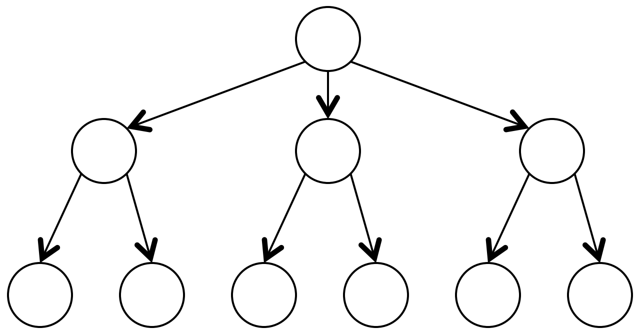
# CS200:
# Recursion and induction

Prichard Ch. 6.1 & 6.3

# Backtracking

- Problem solving technique that involves **moves: guesses** at a solution.
- **Depth First Search:** in case of failure retrace steps and try a new move in a state with still unexplored guesses

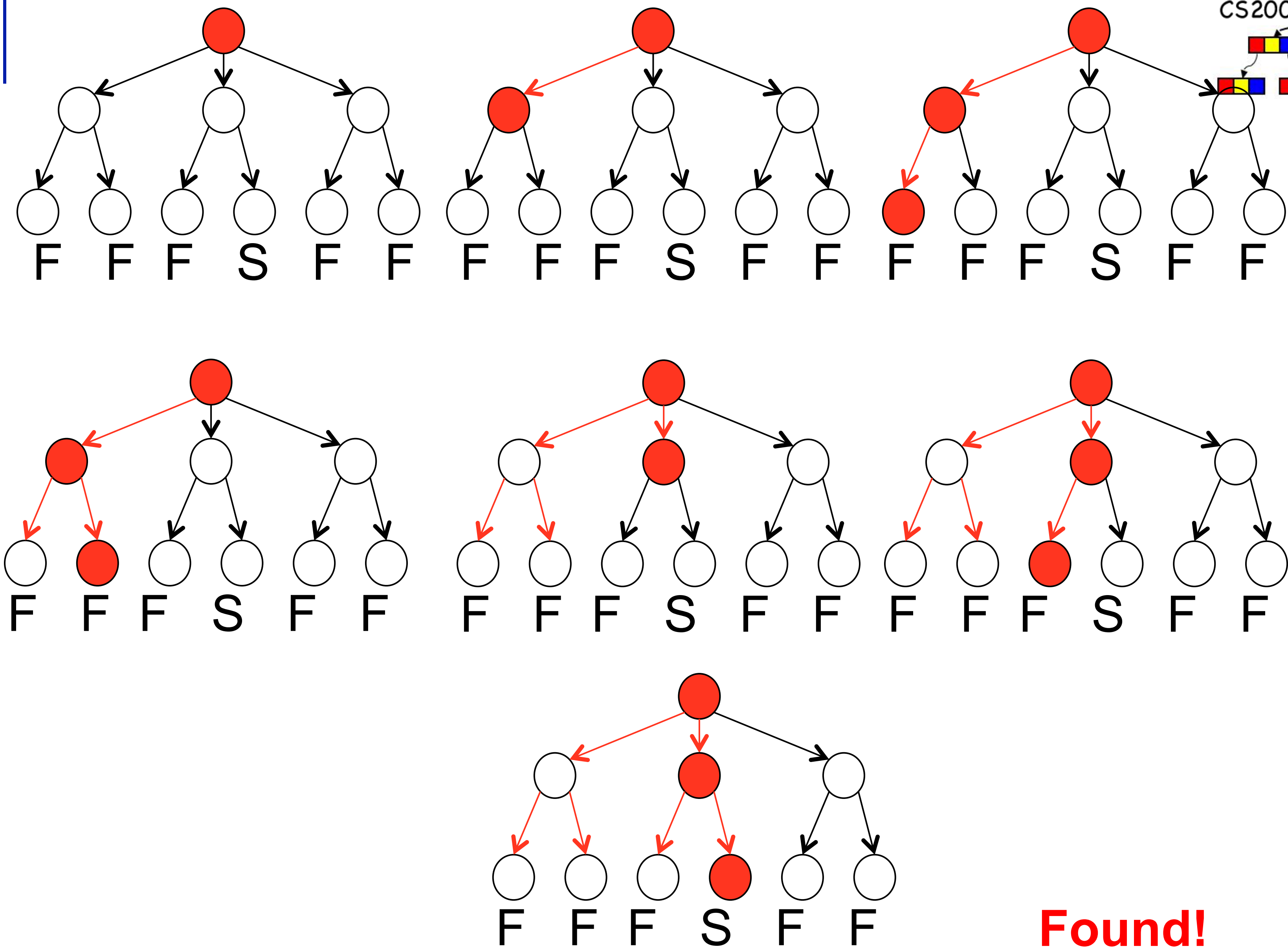
Think of it as walking through a tree shaped state space.



3 guesses here

2 guesses in each state here

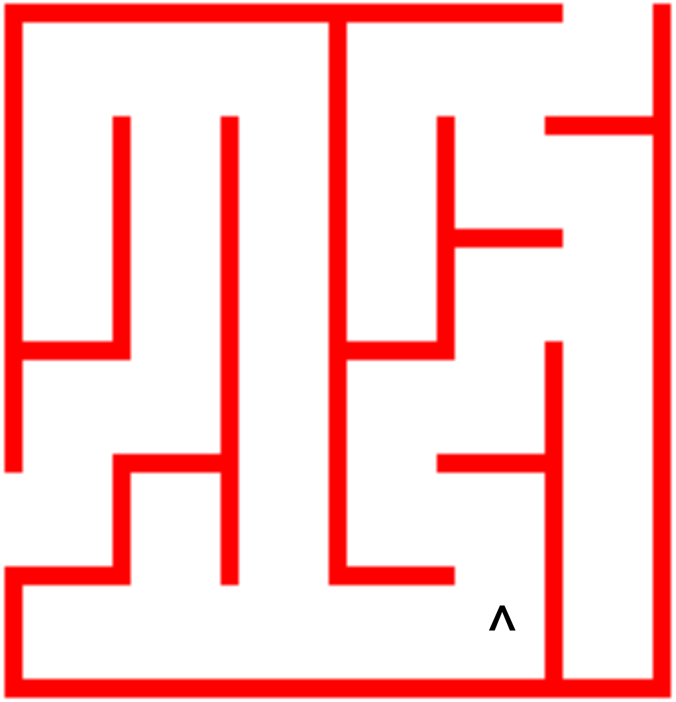
leaf states can fail (F) or succeed (S)




**Found!**

# Depth First Search

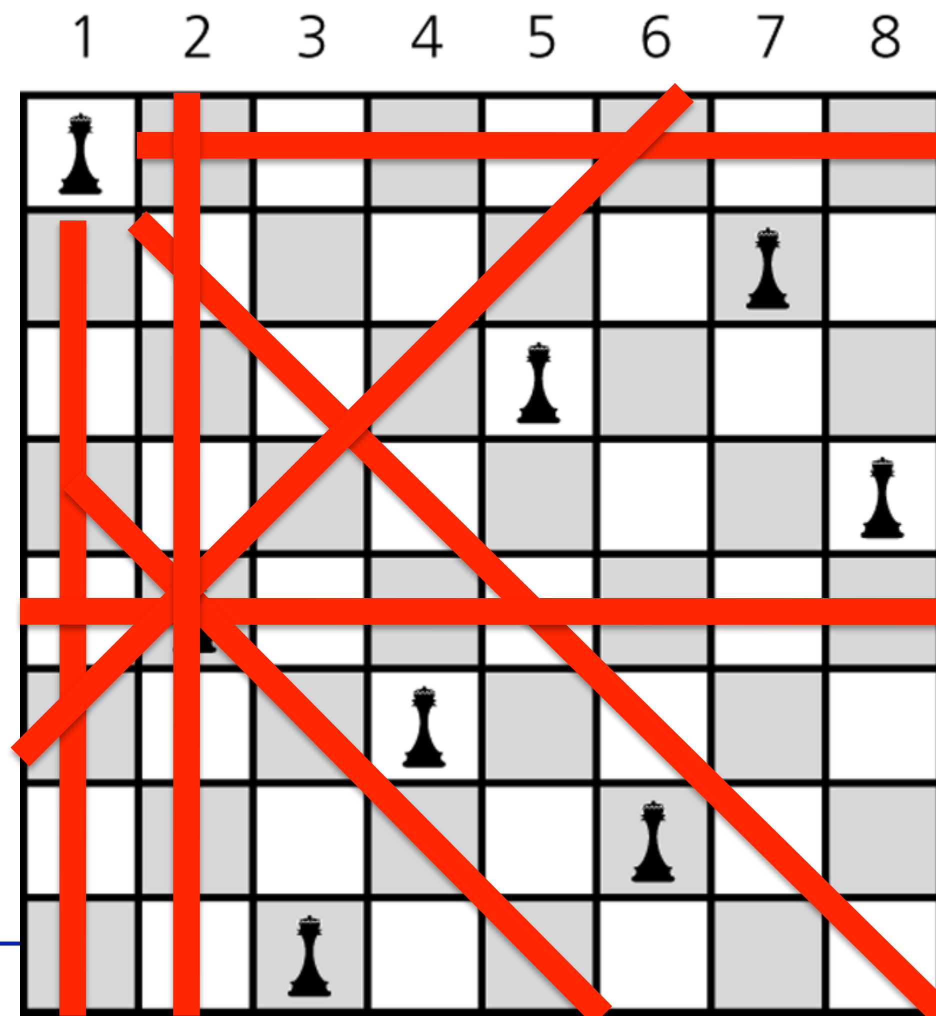
- Looking for a path out of the maze
- Strategy:
  - Prioritize directions:  right, straight or left.
  - At a dead end "backtrack" and try a different direction
- Recursive solution?



# The Eight Queens Problem

Place 8 Queens!

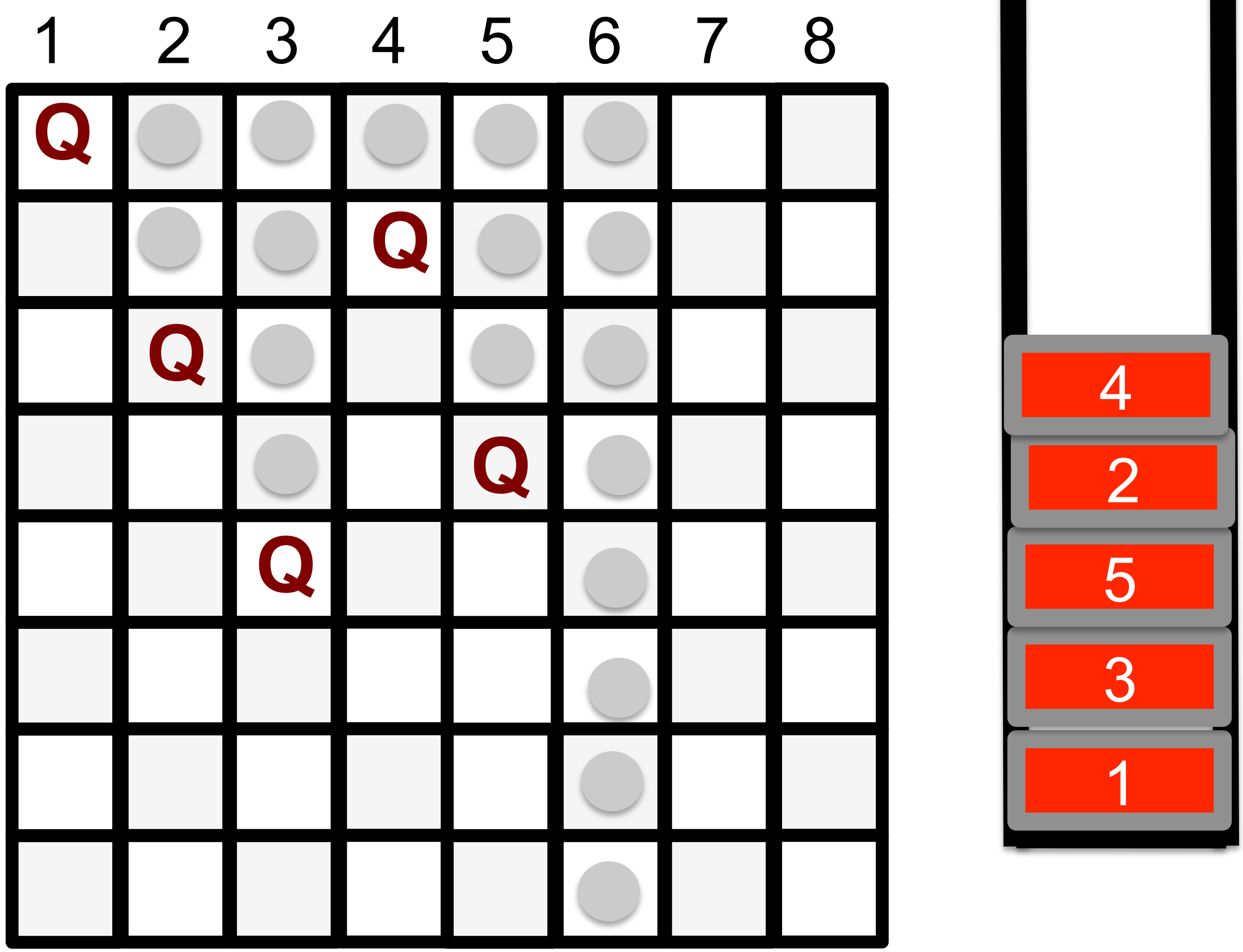
No queen can attack any other queens.



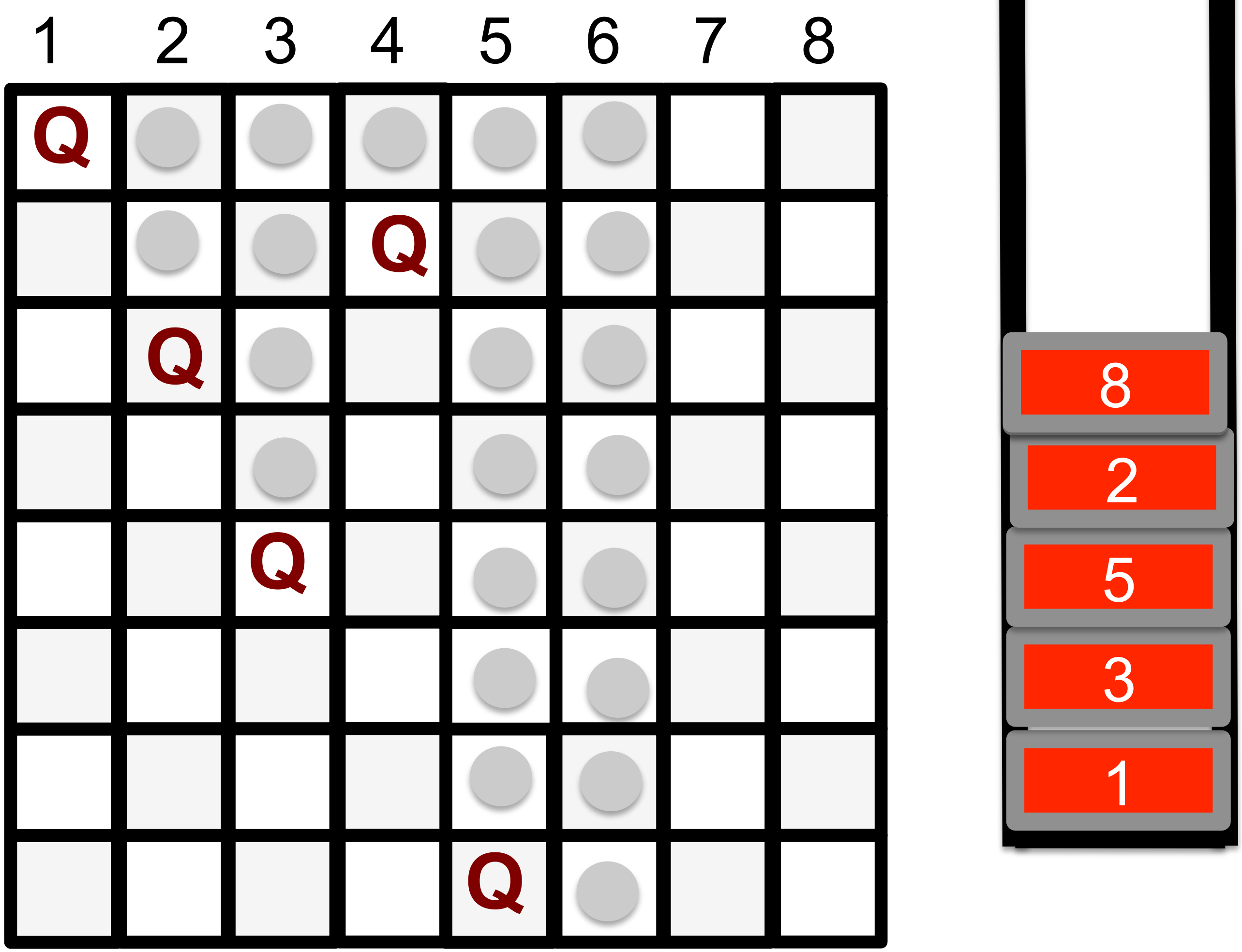
# Solution with recursion and backtracking

```
placeQueen (in currColumn:integer)
if ( currColumn > 8) {
   The problem is solved
} else {
   while (unconsidered squares exist in currColumn and the
          problem is unsolved) {
      Determine if the next square is safe.
      if (such a square exists){
         place a queen in the square
         placeQueens(currColumn+1) // try next column
          if (no queen safe in currColumn+1) {
               remove queen from currColumn
               try the next square in that column
          }
      }
   }
}
```

# Example

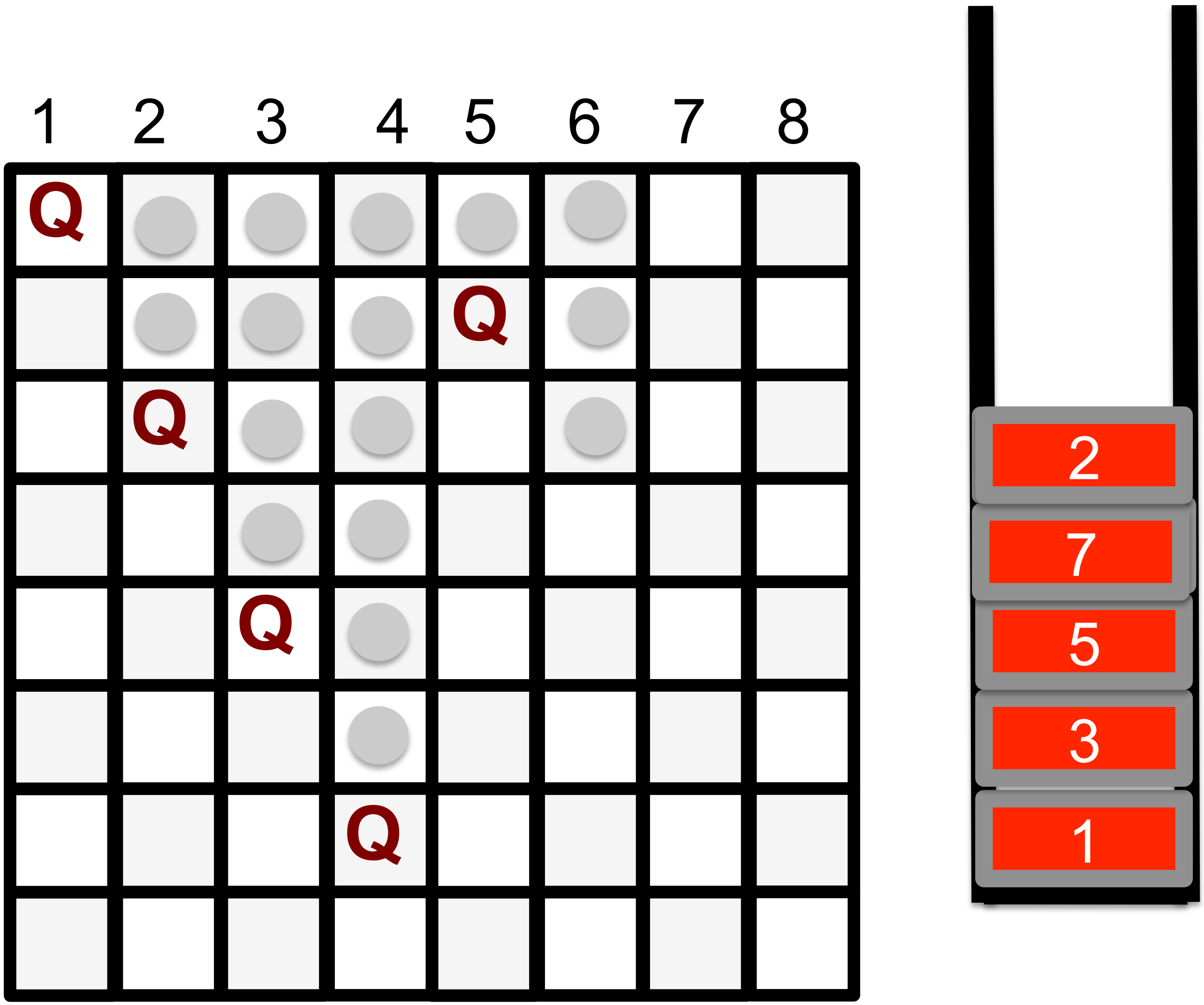


# Hit ‘Dead End’

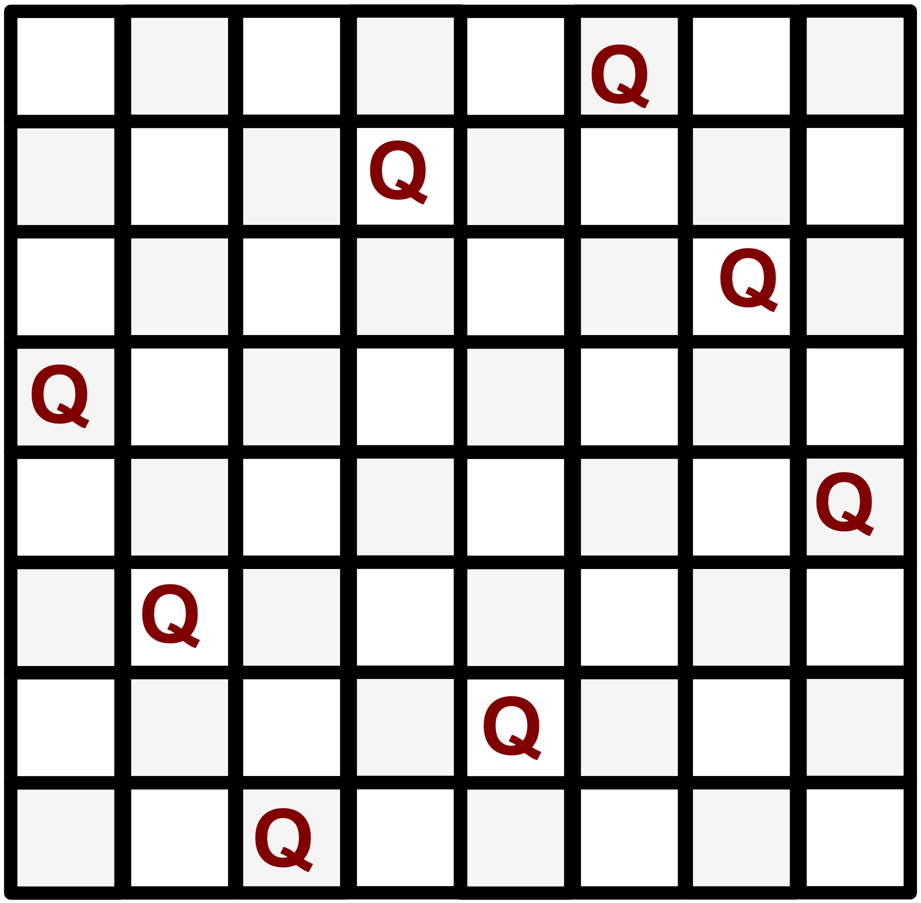


# Backtrack

If you go on, this will fail and you need to back track to col 4



# Backtrack: an 8 queens solution




The only symmetric one

There are 11 more "fundamental" solutions

see:

wikipedia.org/wiki/Eight_queens_puzzle

# Questions

- What is the maximum depth of the  runt time stack for 8 Queens?

- How big could the call tree get?

# Recursion

- Specifies a solution to one or more base cases
- Then demonstrates how to derive the solution to a problem of an arbitrary size
  - From solutions to smaller sized problems.

# Correctness of the Recursive Factorial Method

Specification of the problem
(e.g., Mathematical definition, SW requirements)

Algorithm
(e.g., pseudo code)

Does your algorithm satisfy the specification of the problem?

# Correctness of the Recursive Factorial Method

**Definition of Factorial**

$factorial(n) = n\ (n-1)\ (n-2)\ \ldots\ 1$ for any integer $n > 0$

$factorial(0) = 1$

**Definition of method** *fact(N)*

```
fact (in n: integer): integer
    if (n is 0) {
        return 1
    } else {
        return n* fact(n-1)
    }
```

# Inductive proof fact computes the factorial of its argument.

**Basis step:**

$fact(0) = 1$

**Inductive Step:**

Show that for an arbitrary positive integer $k$,

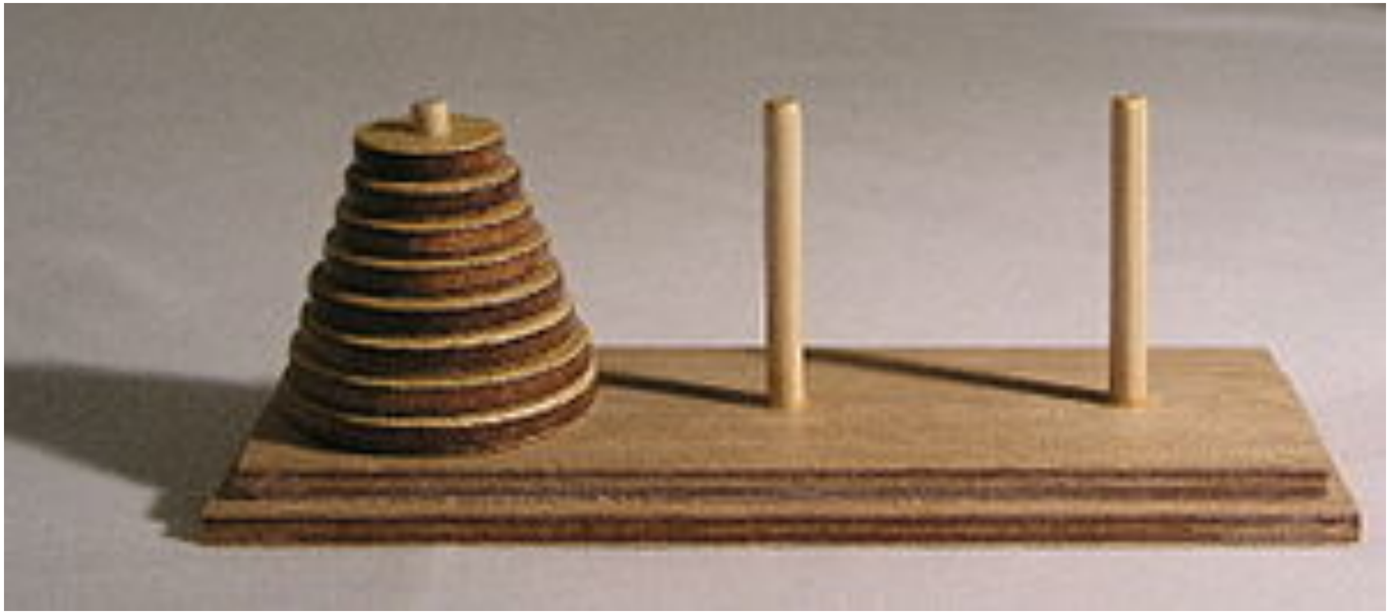
if $fact(k)$ returns $k!$, then

$fact(k+1)$ returns $(k+1)!$

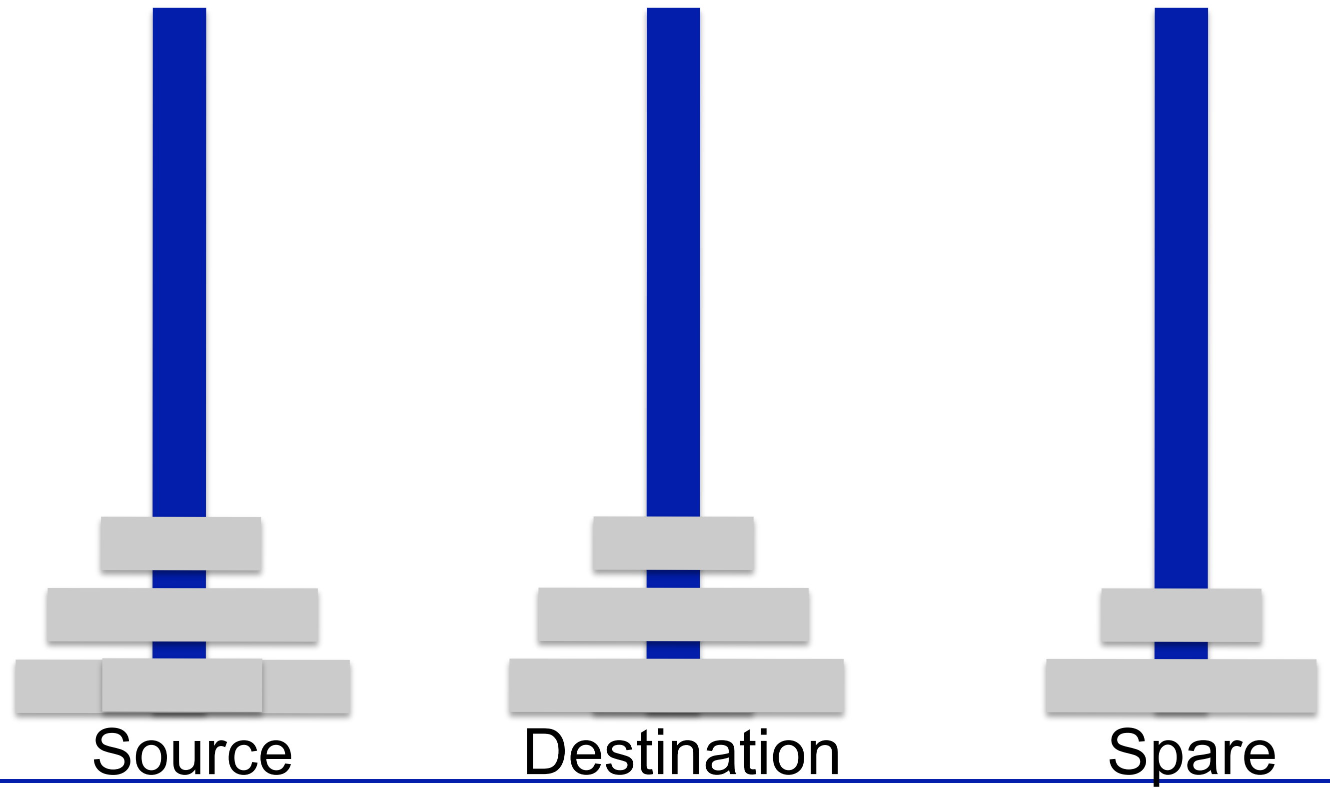
*do it do it*

# The Towers of Hanoi Example

- **Move pile of disks from source to destination**
- **Only one** disk may be moved at a time.
- No disk may be placed on top of a smaller disk.



# States in the Towers of Hanoi



# Recursive Solution

```
// pegs are numbers, via is computed
// number of moves are counted
// empty base case

public void hanoi(int n, int from, int to){
   if (n>0) {
      int via = 6 - from - to;
      hanoi(n-1,from, via);
      System.out.println("move disk " + n + " from " + from + " to " + to);
      hanoi(n-1,via,to);
   }
}
```

let’s run it

# Cost of Towers of Hanoi

- How many moves does *hanoi(n)* make?
- *from the recursive code:*

  $moves(1) = 1$

  $moves(N) = moves(N-1)+1+moves(N-1)$ (if $N>1$)

- By inspection, we can infer that a closed form formula for the number of moves:

  $moves(N) = 2^N - 1$ (for all $N>=1$)

- Can we prove it?

# Proof

- Basis Step
  - Show that the property is true for N = 1.

    $2^1 - 1 = 1$, which is consistent with the recurrence relation's specification that $moves(1) = 1$

- Inductive Step
  - Property is true for an arbitrary $k$ ➔ property is true for $k+1$
  - Assume that the property is true for N = k

    $moves(k) = 2^k-1$
  - Show that the property is true for $N = k + 1$
  - *Do it, do it*

# Proof – cont.

- $moves(k+1) = 2 * moves(k) + 1$

$$= 2 * (2^k - 1) + 1$$

$$= 2 * 2^k - 2 + 1 = \mathbf{2^{k+1} - 1}$$

Therefore the inductive proof is complete.

# $0+1+2\ldots+n = n(n+1)/2 \qquad n=0,1,2\ldots.$

base: $0 = 0*1/2=0$ Check

step: assume: $0+1+2\ldots+k = k(k+1)/2$

show that $0+1+2\ldots+k+ (k+1) = (k+1)(k+2)/2$

$0+1+2\ldots+k+ (k+1) = k(k+1)/2 + (k+1) =$

$k(k+1)/2 + 2(k+1)/2 = k(k+1)/2 + 2(k+1)/2 =$

$(k+2)(k+1)/2 = (k+1)(k+2)/2$

Check