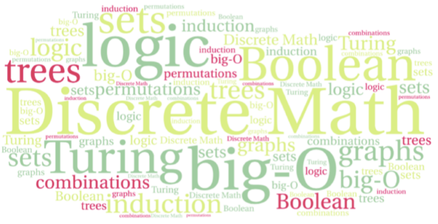
CS 220: Discrete Structures and their Applications

binary relations

zybooks 9.1-9.2



binary relations

A – set of students

B – set of courses

R – pairs (a,b) such that student a is enrolled in course b

R = {(chris, cs220), (mike,cs520),…}

A – set of cities

B – set of US states

R – (a,b) such that city a is in state b

R = {(Denver, CO), (Laramie, WY),…}

binary relations

Definition: A binary relation between two sets A and B is a subset R of A x B.

Recall that $A \times B = \{ (a,b) \mid a \in A \text{ and } b \in B\}$

For $a \in A$ and $b \in B$, the fact that $(a, b) \in R$ is denoted by aRb.

Example:

For $x \in \mathbf{R}$ and $y \in \mathbf{Z}$ define xCy if $|x - y| \leq 1$

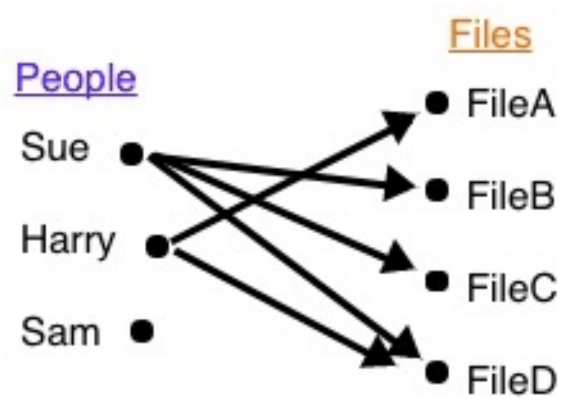
binary relations

a graphical representation of a relation

People = { Sue, Harry, Sam }

Files = { FileA, FileB, FileC, FileD }

Relation A: pAf if person p has access to file f



A = { (Sue, FileB), (Sue, FileC), (Sue, FileD),
(Harry, FileA), (Harry, FileD) }

binary relations

the same binary relation can be represented as a matrix:

People = { Sue, Harry, Sam }

Files = { File A, File B, File C, File D }

Relation A: pAf if person p has access to file f

	File A	File B	File C	File D
Sue	0	1	1	1
Harry	1	0	0	1
Sam	0	0	0	0

A = {(Sue, File B) (Sue, File C) (Sue, File D)
(Harry, File A) (Harry, File D) }

A 2-d array of numbers with |A| rows and |B| columns. Each row corresponds to an element of A and each column corresponds to an element of B. For a ∈ A and b ∈ B, there is a 1 in row a, column b, if aRb and 0 otherwise.

counting binary relations

A binary relation from A to B is a subset of A x B

Given sets A and B with sizes n and m, the number of elements in A x B is nm, and the number of binary relations from A to B is 2^{nm}

WHY?

functions as relations

A function f from A to B assigns an element of B to each element of A.

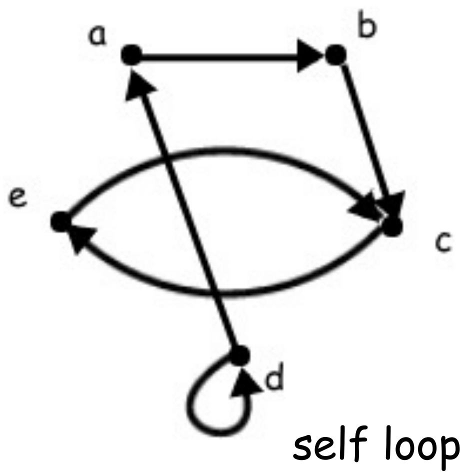
Difference between relations and functions?

binary relations on a set

A binary relation on a set A is a subset of $A \times A$.

The set A is called the domain of the binary relation.

Graphical representation of a binary relation on a set:



$A = \{a, b, c, d, e\}$

$R \subseteq A \times A$

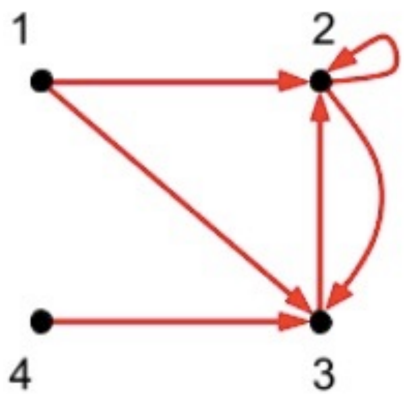
$R = \{(a, b)(b, c)(e, c)(c, e)(d, a)(d, d)\}$

binary relations on a set

Let $A = \{1, 2, 3, 4\}$. Define a relation R on A:

$$R = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (4, 3)\}$$

Can you find the mistakes in the following graphs and matrix representations of this relation?



$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

binary relations on a set

Example: relations on the set of integers

$R_1 = \{(a,b) \mid a \le b\}$

$R_2 = \{(a,b) \mid a > b\}$

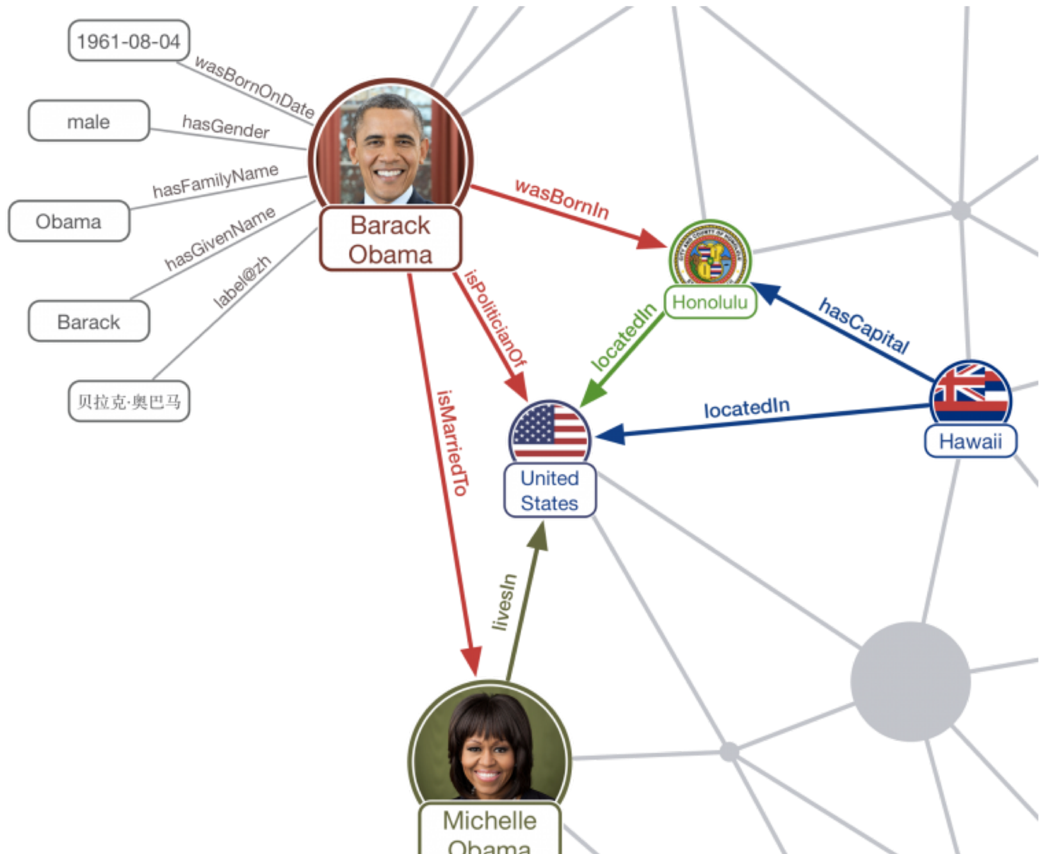
$R_3 = \{(a,b) \mid a = b + 1\}$

application of relations: knowledge graphs

Relations are a way of encoding knowledge.

A knowledge graph is a set of entities (Barak Obama, Hawaii, etc.), relations between those entities (<born_in>).

The relations are used to represent facts e.g. born_in(Barak Obama, Hawaii).



https://www.ambiverse.com/knowledge-graphs-encyclopaedias-for-machines/

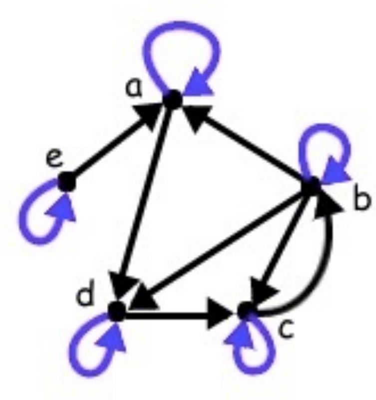
properties of binary relations

Let R be a relation on a set A

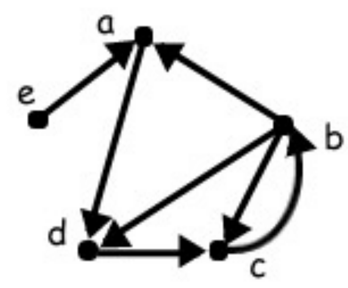
The relation R is reflexive if for every x ∈ A, xRx.

Example: the less-or-equal to relation on the positive integers

The relation R is anti-reflexive if for every x ∈ A, it is not true that xRx.



A = { a, b, c, d, e }



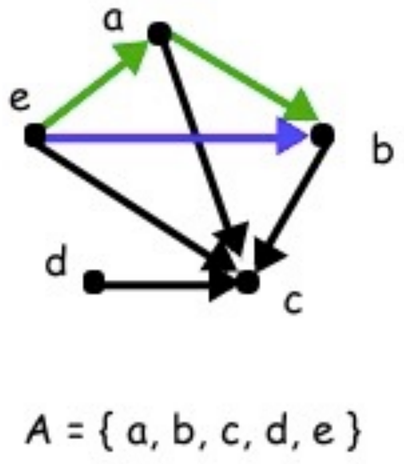
A = { a, b, c, d, e }

properties of binary relations

Let R be a relation on a set A.

The relation R is transitive if for every x,y, z ∈ A, xRy and yRz imply that xRz.

Example: the ancestor relation



properties of binary relations

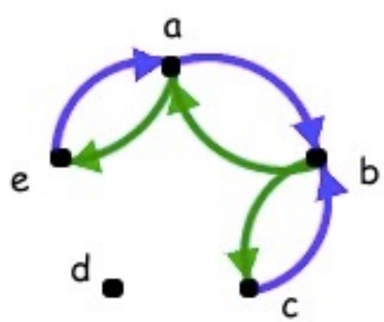
Let R be a relation on a set A.

The relation R is symmetric if for every x,y ∈ A, xRy implies that yRx.

Example:

R = {(a, b) : a,b are actors that have played in the same movie}

The relation R is anti-symmetric if for every x,y ∈ A, xRy and yRx imply that x = y.



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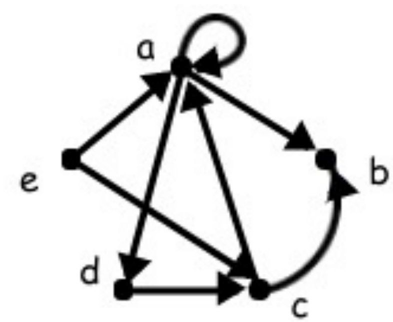

A = { a, b, c, d, e }