

Chapter 2 Bits, Data Types, and Operations

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How do we represent data in a computer?

- ◆ At the lowest level, a computer is an electronic machine.
 - works by controlling the flow of electrons
- ◆ Easy to recognize two conditions:
 1. presence of a voltage – we'll call this state "1"
 2. absence of a voltage – we'll call this state "0"
- ◆ Could base state on *value* of voltage, but control and detection circuits more complex.
 - compare turning on a light switch to measuring or regulating voltage

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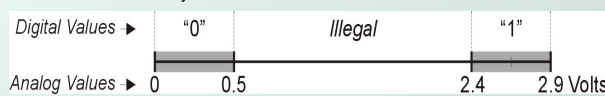
Computer is a binary digital system.

Digital system:

- finite number of symbols

Binary (base two) system:

- has two states: 0 and 1



- ◆ Basic unit of information is the *binary digit*, or *bit*.
- ◆ Values with >2 states require multiple bits.
 - A collection of **two** bits has **four** possible states:
00, 01, 10, 11
 - A collection of **three** bits has **eight** possible states:
000, 001, 010, 011, 100, 101, 110, 111
 - A collection of **n** bits has **2ⁿ** possible states.

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What kinds of data do we need to represent?

- **Numbers** – signed, unsigned, integers, floating point, complex, rational, irrational, ...
- **Text** – characters, strings, ...
- **Logical** – true, false
- **Images** – pixels, colors, shapes, ...
- **Sound** – wave forms
- **Instructions**
- ...
- ◆ Data type:
 - *representation* and *operations* within the computer
- ◆ We'll start with numbers...

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Unsigned Integers

- ◆ Non-positional notation
 - could represent a number ("5") with a string of ones ("11111")
 - problems?
- ◆ Weighted positional notation
 - like decimal numbers: "329"
 - "3" is worth 300, because of its position, while "9" is only worth 9

$$\begin{array}{rcc}
 & \text{most} & \text{least} \\
 & \text{significant} & \text{significant} \\
 329 & & 101 \\
 \begin{array}{c} 10^2 \quad 10^1 \quad 10^0 \\ 3 \times 100 + 2 \times 10 + 9 \times 1 = 329 \end{array} & & \begin{array}{c} 2^2 \quad 2^1 \quad 2^0 \\ 1 \times 4 + 0 \times 2 + 1 \times 1 = 5 \end{array}
 \end{array}$$

Unsigned Integers (cont.)

- ◆ An n -bit unsigned integer represents 2^n values: from 0 to $2^n - 1$.

2^2	2^1	2^0	
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

Unsigned Binary Arithmetic

- ◆ Base-2 addition – just like base-10!
 - add from right to left, propagating carry

$$\begin{array}{r}
 10010 \\
 + 1001 \\
 \hline
 11011
 \end{array}
 \quad
 \begin{array}{r}
 10010 \\
 + 1011 \\
 \hline
 11101
 \end{array}
 \quad
 \begin{array}{r}
 1111 \\
 + 1 \\
 \hline
 10000
 \end{array}$$

$\begin{array}{r}
 10111 \\
 + 111 \\
 \hline
 \end{array}$

Subtraction, multiplication, division: remember integer math!

Signed Integers

- ◆ With n bits, we have 2^n distinct values.
 - assign about half to positive integers (1 through 2^{n-1})
 - assign about half to negative (-2^{n-1} through -1)
 - that leaves two values: one for 0, and one extra
- ◆ Positive integers
 - just like unsigned – zero in *most significant* (MS) bit
 - 00101 = 5**
- ◆ Negative integers
 - sign-magnitude – set sign bit to show negative
 - 10101 = -5**
 - one's complement – flip every bit to represent negative
 - 11010 = -5**
 - in either case, MS bit indicates sign: 0=pos., 1=neg.

Two's Complement

- ◆ Problems with sign-magnitude, 1's complement
 - two representations of zero (+0 and -0)
 - arithmetic circuits are complex
 - ◆ How to add two sign-magnitude numbers?
 - e.g., try 2 + (-3)
 - ◆ How to add to one's complement numbers?
 - e.g., try 4 + (-3)

Two's Complement

- ◆ **Two's complement** representation developed to make circuits easy for arithmetic.
 - for each positive number (X), assign value to its negative (-X), such that $X + (-X) = 0$ with "normal" addition, ignoring carry out

$$\begin{array}{rcl}
 00101 & (5) & \\
 + 11011 & (-5) & \\
 \hline
 00000 & (0) &
 \end{array}
 \qquad
 \begin{array}{rcl}
 01001 & (9) & \\
 + 10991 & (-9) & \\
 \hline
 00000 & (0) &
 \end{array}$$

Two's Complement Representation

- ◆ If number is positive or zero,
 - normal binary representation, zeroes in upper bit(s)
- ◆ If number is negative,
 - start with positive number
 - flip every bit (i.e., take the one's complement)
 - then add one

$$\begin{array}{rcl}
 \curvearrowright 00101 & (5) & \curvearrowright 01001 & (9) \\
 \curvearrowright 11010 & (1's \text{ comp}) & \curvearrowright 10991 & (1's \text{ comp}) \\
 + 1 & & + 1 & \\
 \hline
 11011 & (-5) & 10991 & (-9)
 \end{array}$$

Two's Complement Shortcut

- ◆ To take the two's complement of a number:
 - copy bits from right to left until (and including) first "1"
 - flip remaining bits to the left

$$\begin{array}{rcl}
 \curvearrowright 011010000 & & 011010000 \\
 \curvearrowright 100101111 & (1's \text{ comp}) & \downarrow \text{(flip)} \\
 + 1 & & \downarrow \text{(copy)} \\
 \hline
 100110000 & & 100110000
 \end{array}$$

Two's Complement Signed Integers

- MS bit is sign bit – it has weight -2^{n-1} .
- Range of an n-bit number: -2^{n-1} through $2^{n-1} - 1$.
 - The most negative number has no positive counterpart.

-2^3	2^2	2^1	2^0		-2^3	2^2	2^1	2^0	
0	0	0	0	0	1	0	0	0	-8
0	0	0	1	1	1	0	0	1	-7
0	0	1	0	2	1	0	1	0	-6
0	0	1	1	3	1	0	1	1	-5
0	1	0	0	4	1	1	0	0	-4
0	1	0	1	5	1	1	0	1	-3
0	1	1	0	6	1	1	1	0	-2
0	1	1	1	7	1	1	1	1	-1

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Converting Binary (2's C) to Decimal

- If leading bit is one, take two's complement to get a positive number.
- Add powers of 2 that have "1" in the corresponding bit positions.
- If original number was negative, add a minus sign.

$$\begin{aligned}
 X &= 01101000_{\text{two}} \\
 &= 2^6 + 2^5 + 2^3 = 64 + 32 + 8 \\
 &= 104_{\text{ten}}
 \end{aligned}$$

Assuming 8-bit 2's complement numbers.

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

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More Examples

$$\begin{aligned}
 X &= 00100111_{\text{two}} \\
 &= 2^5 + 2^2 + 2^1 + 2^0 = 32 + 4 + 2 + 1 \\
 &= 39_{\text{ten}}
 \end{aligned}$$

$$\begin{aligned}
 X &= 11100110_{\text{two}} \\
 -X &= 00011010 \\
 &= 2^4 + 2^3 + 2^1 = 16 + 8 + 2 \\
 &= 26_{\text{ten}} \\
 X &= -26_{\text{ten}}
 \end{aligned}$$

Assuming 8-bit 2's complement numbers.

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

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Converting Decimal to Binary (2's C)

- First Method: **Division**
- Find magnitude of decimal number
 - Divide by two – remainder is least significant bit.
 - Keep dividing by two until answer is zero, writing remainders from right to left.
 - Append a zero as the MS bit; for negative, take two's complement.

$$\begin{aligned}
 X &= 104_{\text{ten}} & 104 - 64 &= 40 & \text{bit 6} \\
 & & 40 - 32 &= 8 & \text{bit 5} \\
 & & 8 - 8 &= 0 & \text{bit 3}
 \end{aligned}$$

$$X = 01101000_{\text{two}}$$

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

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Converting Decimal to Binary (2's C)

◆ Second Method: **Subtract Powers of Two**

1. Find magnitude of decimal number.
2. Subtract largest power of two less than or equal to number.
3. Put a one in the corresponding bit position.
4. Keep subtracting until result is zero.
5. Append a zero as MS bit; if original was negative, take two's complement.

	<i>n</i>	2^n
	0	1
	1	2
	2	4
	3	8
	4	16
	5	32
	6	64
	7	128
	8	256
	9	512
	10	1024

$$X = 104_{\text{ten}} \quad \begin{array}{l} 104 - 64 = 40 \quad \text{bit 6} \\ 40 - 32 = 8 \quad \text{bit 5} \\ 8 - 8 = 0 \quad \text{bit 3} \end{array}$$

$$X = 01101000_{\text{two}}$$

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Operations: Arithmetic and Logical

- ◆ Recall: data types include **representation** and **operations**.
- ◆ 2's complement is a good representation for signed integers, now we need arithmetic operations:
 - **Addition (including overflow)**
 - **Subtraction**
 - **Sign Extension**
- ◆ Multiplication and division can be built from these basic operations.
- ◆ Logical operations are also useful:
 - **AND**
 - **OR**
 - **NOT**

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Addition

- ◆ As we've discussed, 2's comp. addition is just binary addition.

- assume all integers have the same number of bits
- ignore carry out
- for now, assume that sum fits in n-bit 2's comp. representation

$$\begin{array}{r} 01101000 \text{ (104)} \\ + 11110000 \text{ (-16)} \\ \hline 01011000 \text{ (98)} \end{array} \quad \begin{array}{r} 11110110 \text{ (-10)} \\ + \quad \quad \quad \text{(-9)} \\ \hline \quad \quad \quad \text{(-19)} \end{array}$$

Assuming 8-bit 2's complement numbers.

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Subtraction

- ◆ Negate second operand, then add.
 - assume all integers have the same number of bits
 - ignore carry out
 - for now, assume that difference fits in n-bit 2's comp. representation

$$\begin{array}{r} 01101000 \text{ (104)} \\ - 00010000 \text{ (16)} \\ \hline 01101000 \text{ (104)} \\ + 11110000 \text{ (-16)} \\ \hline 01011000 \text{ (88)} \end{array} \quad \begin{array}{r} 11110110 \text{ (-10)} \\ - \quad \quad \quad \text{(-9)} \\ \hline 11110110 \text{ (-10)} \\ + \quad \quad \quad \text{(9)} \\ \hline \quad \quad \quad \text{(-1)} \end{array}$$

Assuming 8-bit 2's complement numbers.

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Sign Extension

- ◆ To add two numbers, we must represent them with the same number of bits.

- ◆ If we just pad with zeroes on the left:

4-bit	8-bit
0100 (4)	00000100 (still 4)
1100 (-4)	00001100 (12, not -4)

- ◆ Instead, replicate the MS bit -- the sign bit:

4-bit	8-bit
0100 (4)	00000100 (still 4)
1100 (-4)	11111100 (still -4)

Overflow

- ◆ If operands are too big, then sum cannot be represented as an n -bit 2's comp number.

01000 (8)	11000 (-8)
+ 01001 (9)	+ 10111 (-9)
10001 (-15)	01111 (+15)

- ◆ We have overflow if:

- signs of both operands are the same, and
- sign of sum is different.

- ◆ Another test -- easy for hardware:

- carry into MS bit does not equal carry out

Logical Operations

- ◆ Operations on logical TRUE or FALSE

- two states -- takes one bit to represent: TRUE=1, FALSE=0

A	B	A AND B	A	B	A OR B	A	NOT A
0	0	0	0	0	0	0	1
0	1	0	0	1	1	1	0
1	0	0	1	0	1		
1	1	1	1	1	1		

- ◆ View n -bit number as a collection of n logical values

- operation applied to each bit independently

Examples of Logical Operations

- ◆ AND

- useful for clearing bits

- ◆ AND with zero = 0
- ◆ AND with one = no change

	11000101
AND	00001111
	00000101

- ◆ OR

- useful for setting bits

- ◆ OR with zero = no change
- ◆ OR with one = 1

	11000101
OR	00001111
	11001111

- ◆ NOT

- unary operation -- one argument
- flips every bit

	11000101
NOT	00111010

Hexadecimal Notation

- It is often convenient to write binary (base-2) numbers in hexadecimal (base-16) instead.

- fewer digits - four bits per hex digit
- less error prone - no long string of 1's and 0's

Binary	Hex	Decimal	Binary	Hex	Decimal
0000	0	0	1000	8	8
0001	1	1	1001	9	9
0010	2	2	1010	A	10
0011	3	3	1011	B	11
0100	4	4	1100	C	12
0101	5	5	1101	D	13
0110	6	6	1110	E	14
0111	7	7	1111	F	15

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Converting from Binary to Hexadecimal

- Every four bits is a hex digit.

- start grouping from right-hand side

011 1010 1000 1111 0100 1101 0111

↓ ↓ ↓ ↓ ↓ ↓ ↓

3 A 8 F 4 D 7

*This is not a new machine representation,
just a convenient way to write the number.*

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Fractions: Fixed-Point

- How can we represent fractions?
 - Use a "binary point" to separate positive from negative powers of two -- just like "decimal point."
 - 2's comp addition and subtraction still work (if binary points are aligned)

$2^{-1} = 0.5$
 $2^{-2} = 0.25$
 $2^{-3} = 0.125$

00101000.101 (40.625)
 + 11111110.110 (-1.25)
 00100111.011 (39.375)

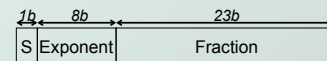
No new operations -- same as integer arithmetic.

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Very Large and Very Small: Floating-Point

- Large values: 6.023×10^{23} -- requires 79 bits
- Small values: 6.626×10^{-34} -- requires >110 bits
- Use equivalent of "scientific notation": $F \times 2^E$
- Must have F (fraction), E (exponent), and sign.
- IEEE 754 Floating-Point Standard (32-bits):



$$N = (-1)^S \times 1.\text{fraction} \times 2^{\text{exponent} - 127}, \quad 1 \leq \text{exponent} \leq 254$$

$$N = (-1)^S \times 0.\text{fraction} \times 2^{-126}, \quad \text{exponent} = 0$$

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Floating Point Example

- Single-precision IEEE floating point number:

1 01111110 100000000000000000000000

↑ ↑ ↑
sign exponent fraction

- Sign is 1 – number is negative.
- Exponent field is 01111110 = 126 (decimal).
- Fraction is 1.100000000000... = 1.5 (decimal).

Value = $-1.5 \times 2^{(126-127)} = -1.5 \times 2^{-1} = -0.75$

Floating-Point Operations

- Will regular 2's complement arithmetic work for Floating Point numbers?
- (Hint: In decimal, how do we compute $3.07 \times 10^{12} + 9.11 \times 10^8$?)

Text: ASCII Characters

- ASCII: Maps 128 characters to 7-bit code.

- printable and non-printable (ESC, DEL, ...) characters

00	nl	10	dl	20	sp	30	0	40	@	50	P	60	`	70	p
01	soh	11	dc1	21	!	31	1	41	A	51	Q	61	a	71	q
02	stx	12	dc2	22	"	32	2	42	B	52	R	62	b	72	r
03	etx	13	dc3	23	#	33	3	43	C	53	S	63	c	73	s
04	eot	14	dc4	24	\$	34	4	44	D	54	T	64	d	74	t
05	enq	15	nak	25	%	35	5	45	E	55	U	65	e	75	u
06	ack	16	syn	26	&	36	6	46	F	56	V	66	f	76	v
07	bel	17	etb	27	'	37	7	47	G	57	W	67	g	77	w
08	bs	18	can	28	(38	8	48	H	58	X	68	h	78	x
09	ht	19	em	29)	39	9	49	I	59	Y	69	i	79	y
0a	nl	1a	sub	2a	*	3a	:	4a	J	5a	Z	6a	j	7a	z
0b	vt	1b	esc	2b	+	3b	;	4b	K	5b	[6b	k	7b	{
0c	np	1c	fs	2c	,	3c	<	4c	L	5c	\	6c	l	7c	
0d	cr	1d	gs	2d	-	3d	=	4d	M	5d]	6d	m	7d	}
0e	so	1e	rs	2e	.	3e	>	4e	N	5e	^	6e	n	7e	~
0f	si	1f	us	2f	/	3f	?	4f	O	5f	_	6f	o	7f	del

Interesting Properties of ASCII Code

- What is relationship between a decimal digit ('0', '1', ...) and its ASCII code?
- What is the difference between an upper-case letter ('A', 'B', ...) and its lower-case equivalent ('a', 'b', ...)?
- Given two ASCII characters, how do we tell which comes first in alphabetical order?
- Are 128 characters enough?
(<http://www.unicode.org/>)

No new operations needed for ASCII codes –
integer arithmetic and logic are sufficient.

Other Data Types

- ◆ Text strings
 - array of characters, terminated with null character ('\0')
 - typically, no hardware support
- ◆ Image
 - array of pixels
 - ◆ monochrome: one bit (1/0 = black/white)
 - ◆ color: red, green, blue (RGB) components
 - ◆ other properties: transparency
 - hardware support:
 - ◆ typically none, in general-purpose processors
 - ◆ MMX -- multiple 8-bit operations on 32-bit word
- ◆ Sound
 - sequence of fixed-point numbers

LC-3 Data Types

- ◆ Some data types are supported directly by the instruction set architecture.
- ◆ For LC-3, there is only one hardware-supported data type:
 - 16-bit 2's complement signed integer
 - Operations: ADD, AND, NOT
- ◆ Other data types are supported by interpreting 16-bit values as logical, text, fixed-point, floating-point, etc., in the software that we write.