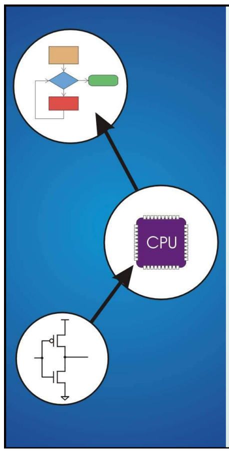


Chapter 17
Recursion

Original slides from Gregory Byrd, North Carolina State University

Modified slides by Chris Wilcox, Colorado State University

What is Recursion?

- A **recursive function** is one that solves its task by **calling itself** on smaller pieces of data.
  - Similar to recurrence function in mathematics.
  - Like iteration -- can be used interchangeably; sometimes recursion results in a simpler solution.

Example: Running sum ($\sum_{1}^{n} i$)

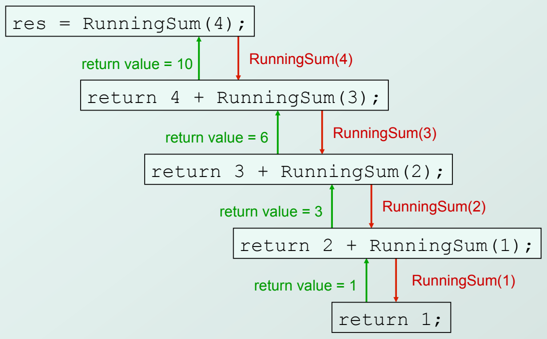
Mathematical Definition:

RunningSum(1) = 1
RunningSum(n) =
n + RunningSum(n-1)

Recursive Function:

```
int RunningSum(int n) {
  if (n == 1)
    return 1;
  else
    return n + RunningSum(n-1);
}
```

Executing RunningSum



```
res = RunningSum(4);
return 4 + RunningSum(3);
return 3 + RunningSum(2);
return 2 + RunningSum(1);
return 1;
```

High-Level Example: Binary Search

- Given a sorted set of exams, in alphabetical order, find the exam for a particular student.

1. Look at the exam **halfway** through the pile.
2. If it matches the name, we're done; if it does not match, then...

3a. If the name is greater (alphabetically), then **search the upper half** of the stack.

3b. If the name is less than the halfway point, then **search the lower half** of the stack.

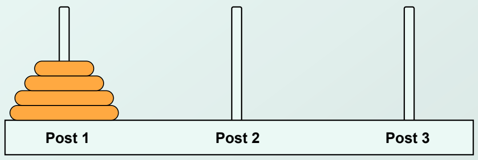
Binary Search: Pseudocode

- Pseudocode is a way to describe algorithms without completely coding them in C.

```
FindExam(studentName, start, end) {
  halfwayPoint = (end + start)/2;
  if (end < start)
    ExamNotFound();  /* exam not in stack */
  else if (studentName == NameOfExam(halfwayPoint))
    ExamFound(halfwayPoint); /* found exam! */
  else if (studentName < NameOfExam(halfwayPoint))
    /* search lower half */
    FindExam(studentName, start, halfwayPoint-1)
  else
    /* search upper half */
    FindExam(studentName, halfwayPoint + 1, end);
}
```

High-Level Example: Towers of Hanoi

- **Task:** Move all disks from one post to another post.



Rules:

(1) Can only move one disk at a time.

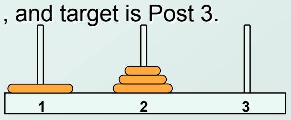
(2) Cannot put larger disk on top of a smaller disk.

(3) May use third post for temporary storage.

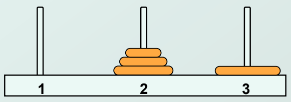
Task Decomposition

- Disks start on Post 1, and target is Post 3.

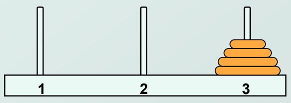
1. Move top n-1 disks to Post 2.



2. Move largest disk to Post 3.



3. Move n-1 disks from Post 2 to Post 3.



Task Decomposition (cont.)

- Task 1 is really the **same problem**, with fewer disks and a different target post.
  - "Move n-1 disks from Post 1 to Post 2."
- And Task 3 is also the **same problem**, with fewer disks and different starting and target posts.
  - "Move n-1 disks from Post 2 to Post 3."
- So this is a **recursive** algorithm.
  - The terminal case is moving the smallest disk -- can move directly without using third post.
  - Number disks from 1 (smallest) to n (largest).

Towers of Hanoi: Pseudocode

```
MoveDisk(diskNumber, startPost, endPost, midPost) {
  if (diskNumber > 1) {
    /* Move top n-1 disks to mid post */
    MoveDisk(diskNumber-1, startPost, midPost, endPost)
    printf("Move disk number %d from %d to %d.\n",
             diskNumber, startPost, endPost);

    /* Move n-1 disks from mid post to end post */
    MoveDisk(diskNumber-1, midPost, endPost, startPost);
  }
  else
    printf("Move disk number 1 from %d to %d.\n",
             startPost, endPost);
}
```

Detailed Example: Fibonacci Numbers

- Mathematical Definition:

$$f(n) = f(n-1) + f(n-2)$$

$$f(1) = 1$$

$$f(0) = 1$$

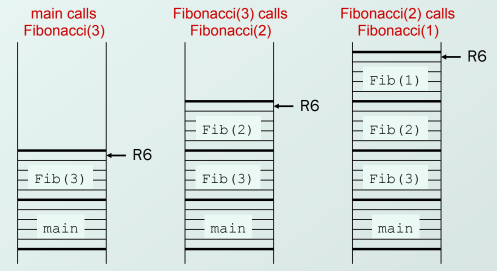
- In other words, the n-th Fibonacci number is the sum of the previous two Fibonacci numbers.

Fibonacci: C Code

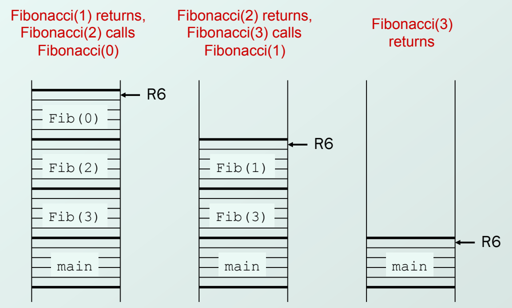
```
int Fibonacci(int n)
{
  if ((n == 0) || (n == 1))
    return 1;
  else
    return Fibonacci(n-1) + Fibonacci(n-2);
}
```

Activation Records

- Whenever Fibonacci is invoked, a new activation record is pushed onto the stack.



Activation Records (cont.)


Fibonacci(1) returns, Fibonacci(2) calls Fibonacci(0)
Fibonacci(2) returns, Fibonacci(3) calls Fibonacci(1)
Fibonacci(3) returns
R6
Fib(0)
Fib(2)
Fib(3)
main
R6
Fib(1)
Fib(3)
main
R6
main

Tracing the Function Calls

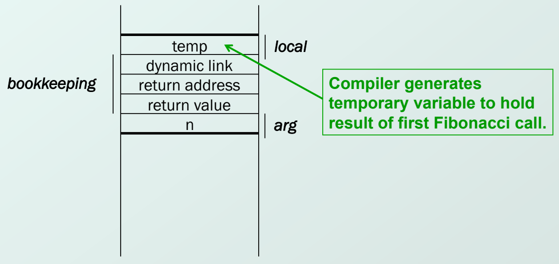
- If we are debugging this program, we might want to trace all the calls of Fibonacci.
  - Note: A trace will also contain the arguments passed into the function.
- For Fibonacci(3), a trace looks like:

```
Fibonacci(3)
Fibonacci(2)
Fibonacci(1)
Fibonacci(0)
Fibonacci(1)
```

- What would trace of Fibonacci(4) look like?

Fibonacci: LC-3 Code

Activation Record


temp
dynamic link
return address
return value
n
local
bookkeeping
arg
Compiler generates temporary variable to hold result of first Fibonacci call.

LC-2 Code (part 1 of 3)

```
Fibonacci

    ADD  R6, R6, #-2 ; skip ret val, push ret addr
    STR  R7, R6, #0
    ADD  R6, R6, #-1  ; push dynamic link
    STR  R5, R6, #0
    ADD  R5, R6, #-1  ; set frame pointer
    ADD  R6, R6, #-2  ; space for locals and temps

    LDR  R0, R5, #4   ; load n
    BRz  FIB_BASE     ; check for terminal cases
    ADD  R0, R0, #-1
    BRz  FIB_BASE
```

LC-3 Code (part 2 of 3)

```
LDR  R0, R5, #4   ; read parameter n
ADD  R0, R0, #-1  ; calculate n-1
ADD  R6, R6, #-1  ; push n-1
STR  R0, R6, #0
JSR  Fibonacci    ; call self

LDR  R0, R6, #0   ; pop return value
ADD  R6, R6, #1
STR  R0, R5, #-1  ; store in temp
LDR  R0, R5, #4   ; read parameter n
ADD  R0, R0, #-2  ; calculate n-2
ADD  R6, R6, #-1  ; push n-2
STR  R0, R6, #0
JSR  Fibonacci    ; call self
```

LC-3 Code (part 3 of 3)

```
          LDR  R0, R6, #0   ; pop return value
          ADD  R6, R6, #1
          LDR  R1, R5, #-1  ; read temp
          ADD  R0, R0, R1   ; Fib(n-1) + Fib(n-2)
          BRnzp  FIB_END    ; all done

FIB_BASE  AND  R0, R0, #0   ; base case – return 1
          ADD  R0, R0, #1

FIB_END   STR  R0, R5, #3   ; write return value (R0)
          ADD  R6, R5, #1   ; pop local variables
          LDR  R5, R6, #0   ; pop dynamic link
          ADD  R6, R6, #1
          LDR  R7, R6, #0   ; pop return address
          ADD  R6, R6, #1
          RET
```

A Final C Example: Printing an Integer

- Recursively converts an unsigned integer as a string of ASCII characters.
  - If integer <10, convert to char and print.
  - else, call self on first (n-1) digits and then print last digit.

```
void IntToAscii(int num) {
  int prefix, currDigit;
  if (num < 10)
    putchar(num + '0');    /* print number */
  else {
    prefix = num / 10;     /* previous digits */
    digit = num % 10;      /* current digit */
    IntToAscii(prefix);    /* recursive call */
    putchar(digit + '0'); /* print digit */
  }
}
```

Trace of IntToAscii

- Calling IntToAscii with parameter 12345:

```
IntToAscii(12345)
 IntToAscii(1234)
  IntToAscii(123)
   IntToAscii(12)
    IntToAscii(1)
    putchar('1')
   putchar('2')
  putchar('3')
 putchar('4')
putchar('5')
```