

CS 410 Midterm 1  
Fall 2012

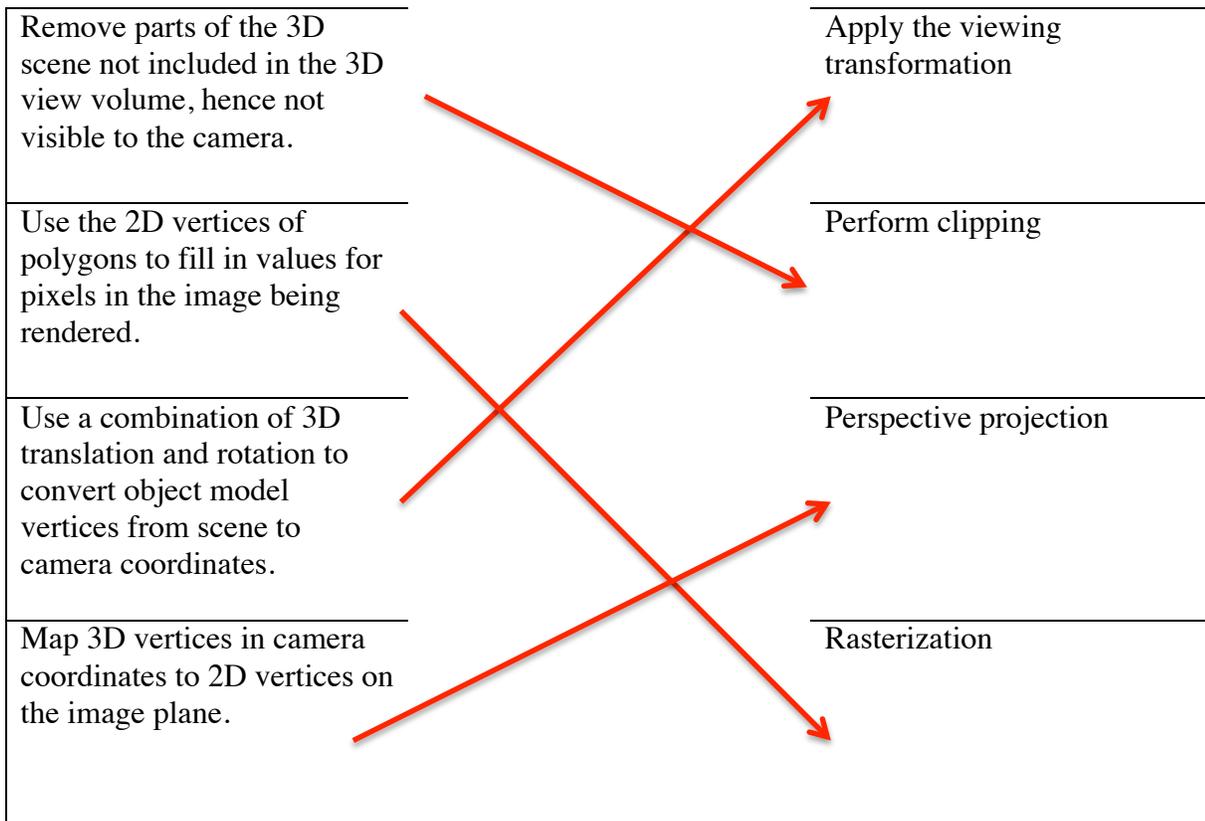
NAME \_\_\_\_\_ **ANSWER KEY** \_\_\_\_\_

EID \_\_\_\_\_

Question	Max Points	Points
1	6	
2	6	
3	6	
4	3	
5	3	
6	3	
7	3	
8	3	
9	3	
10	5	
11	9	
12	7	
13	5	
14	9	
15	5	
16	6	
17	5	
18	3	
19	4	
20	6	
TOTAL	100	

Question 1: Steps in The Projection Pipeline:. (6 Points)

Match the descriptions below to the steps taken when rendering a 3D scene using the perspective projection pipeline approach. Draw an arrow to connect up matches.



Question 2: Convex Polygons (6 Points)

In the attached figure are the matrix specifications for the vertices of four polygons, labeled A through D. The points are specified in column format,  $(x, y, z)$ . This question is not using homogenous coordinates. The vertices are specified in order. One or more of these polygons is not convex. Select the option below indicating the labels of all the polygons that are not convex.

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$C = \begin{bmatrix} 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 2 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 2 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

1. A
2. B
3. C
4. D
5. A, B
6. A, C
7. A, D
8. B, C
9. B, D
10. C, D
11. A, B, C
12. A, B, D
13. A, C, D
14. B, C, D
15. A, B, C, D

Question 3: Rotation about a Point (6 Points)

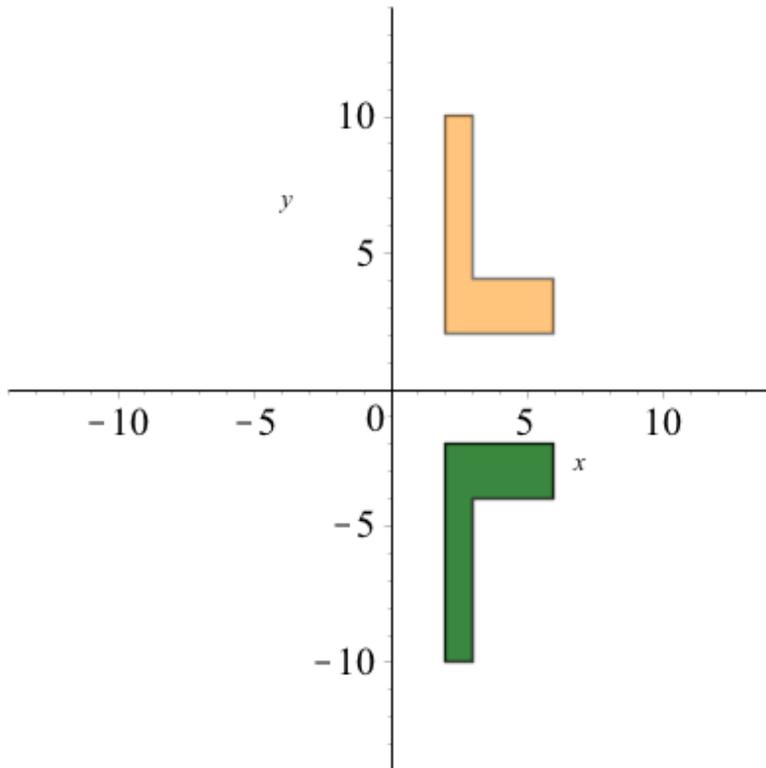
In order to rotate points P in one reference frame to points Q in a new reference frame such that the points are rotated by 45 degrees about the point (10, 10) three of the following four matrices must be applied in the correct order. Below, select the correct ordering.

$$A = \begin{vmatrix} 1 & 0 & -10 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{vmatrix} \quad B = \begin{vmatrix} 1 & 0 & 10 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{vmatrix}$$
$$C = \begin{vmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad D = \begin{vmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

1.  $Q = A C B P$
2.  $Q = B C A P$
3.  $Q = A D B P$
4.  $Q = B D A P$

Question 4: 2D Transformations Example 1 (3 Points)

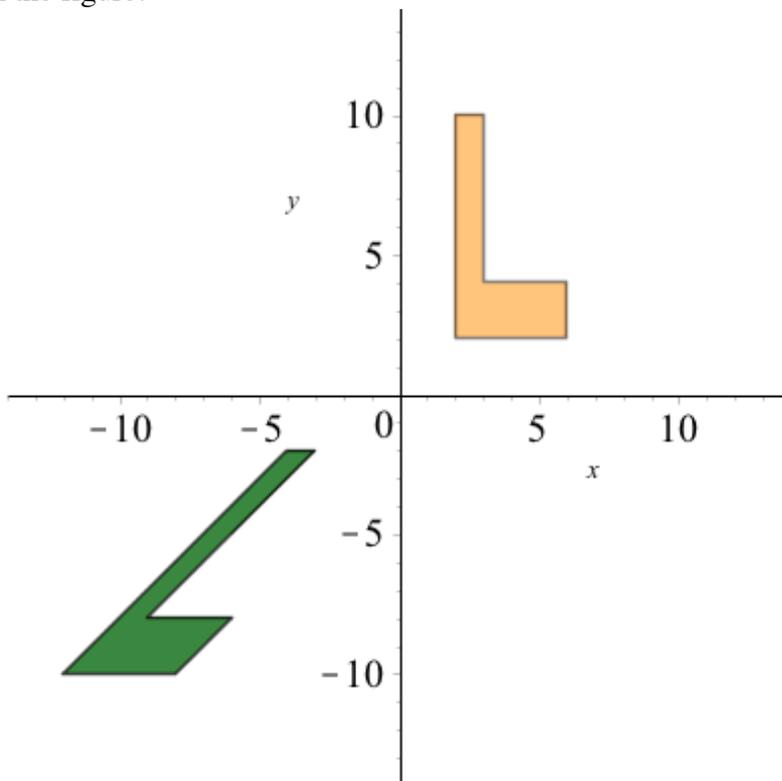
Here is a 2D figure before and after (green) a 2D transformation has been applied. Select the transformation or combination of transformations below that are the best description of what you see in the figure.



1. Rotation
2. Translation
3. Scale
4. Shear
- 5. Flip (Reflect)**
6. Rotation and Translation
7. Scale and Translation
8. Shear and Translation
9. Flip and Translation
10. Rotation, Translation and Scale

Question 5: 2D Transformations Example 2 (3 Points)

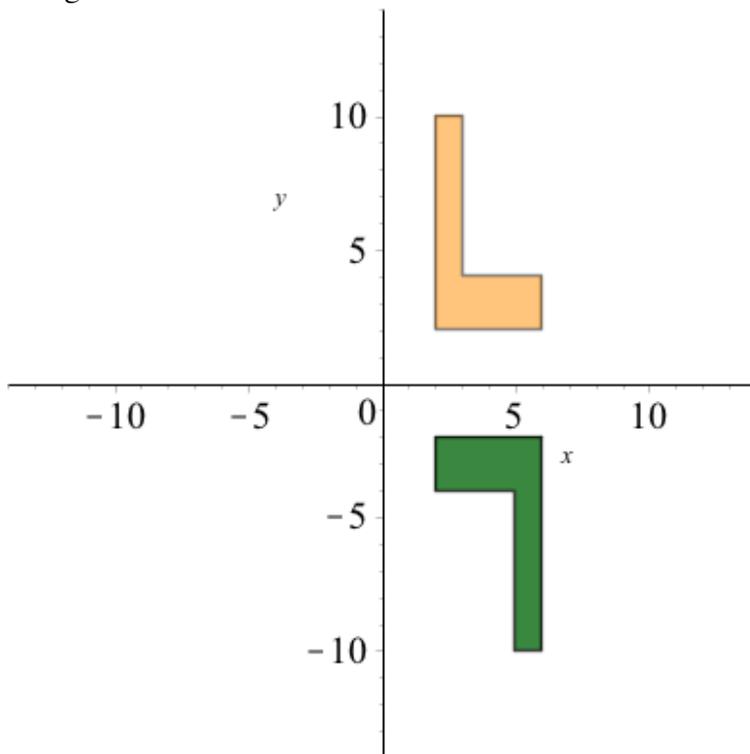
Here is a 2D figure before and after (green) a 2D transformation has been applied. Select the transformation or combination of transformations below that are the best description of what you see in the figure.



1. Rotation
2. Translation
3. Scale
4. Shear
5. Flip (Reflect)
6. Rotation and Translation
7. Scale and Translation
8. **Shear and Translation**
9. Flip and Translation
10. Rotation, Translation and Scale

Question 6: 2D Transformations Example 3 (3 Points)

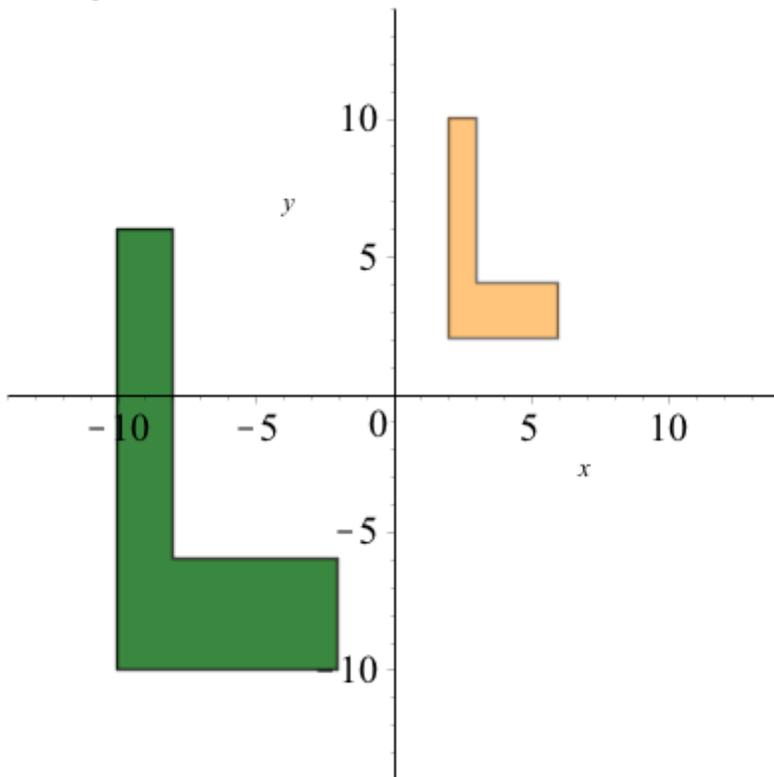
Here is a 2D figure before and after (green) a 2D transformation has been applied. Select the transformation or combination of transformations below that are the best description of what you see in the figure.



1. Rotation
2. Translation
3. Scale
4. Shear
5. Flip (Reflect)
- 6. Rotation and Translation**
7. Scale and Translation
8. Shear and Translation
9. Flip and Translation
10. Rotation, Translation and Scale

Question 7: 2D Transformations Example 4 (3 Points)

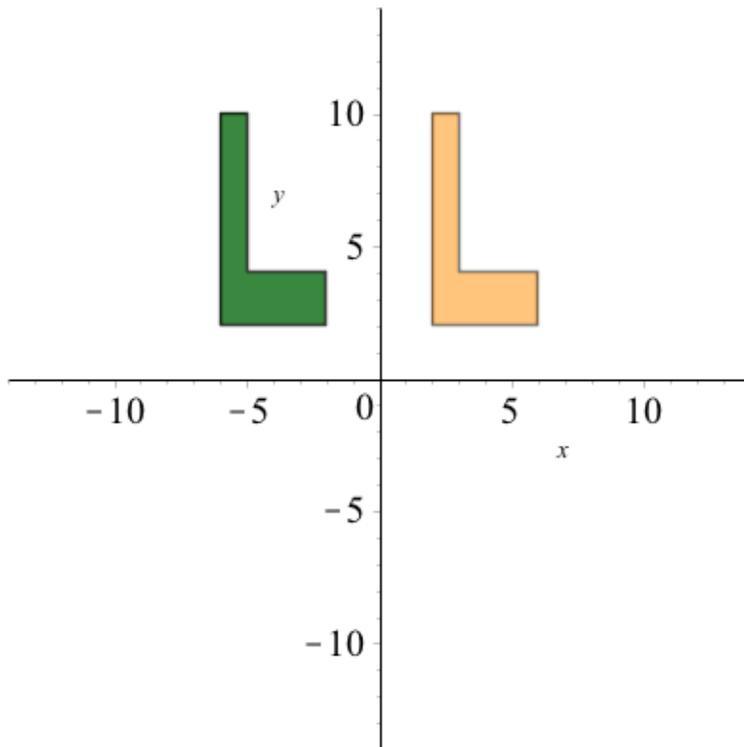
Here is a 2D figure before and after (green) a 2D transformation has been applied. Select the transformation or combination of transformations below that are the best description of what you see in the figure.



1. Rotation
2. Translation
3. Scale
4. Shear
5. Flip (Reflect)
6. Rotation and Translation
- 7. Scale and Translation**
8. Shear and Translation
9. Flip and Translation
10. Rotation, Translation and Scale

Question 8: 2D Transformations Example 5 (3 Points)

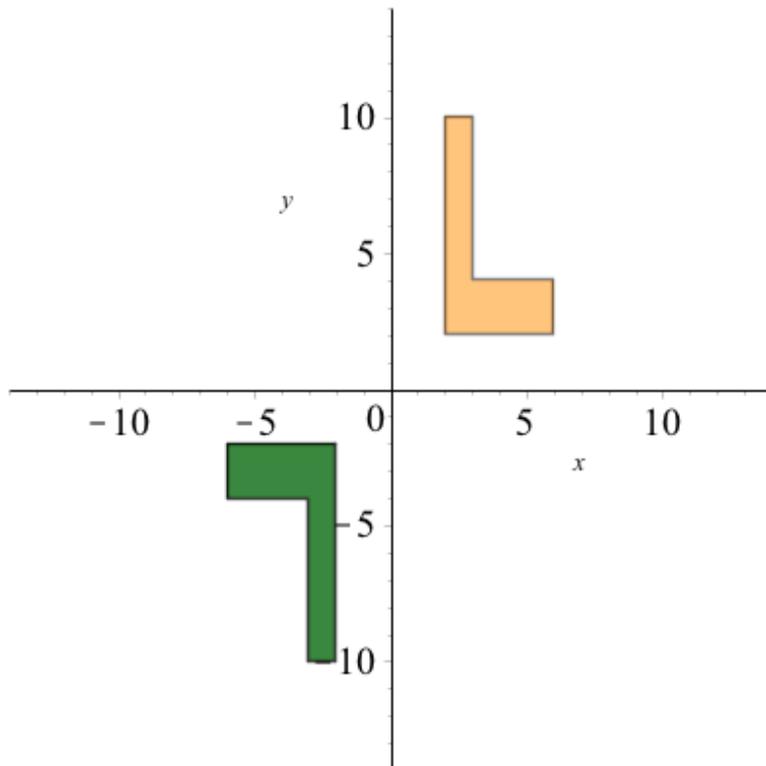
Here is a 2D figure before and after (green) a 2D transformation has been applied. Select the transformation or combination of transformations below that are the best description of what you see in the figure.



1. Rotation
- 2. Translation**
3. Scale
4. Shear
5. Flip (Reflect)
6. Rotation and Translation
7. Scale and Translation
8. Shear and Translation
9. Flip and Translation
10. Rotation, Translation and Scale

Question 9: 2D Transformations Example 6 (3 Points)

Here is a 2D figure before and after (green) a 2D transformation has been applied. Select the transformation or combination of transformations below that are the best description of what you see in the figure.



1. **Rotation**
2. Translation
3. Scale
4. Shear
5. Flip (Reflect)
6. Rotation and Translation
7. Scale and Translation
8. Shear and Translation
9. Flip and Translation
10. Rotation, Translation and Scale

Question 10: Non-Orthographic Basis Vectors (5 Points)

Consider the following pair of basis vectors and amounts alpha and beta of movement along each basis vector. Is it possible to express the vector  $P = (4, 3)$ , and if so, what are the values for alpha and beta. Note  $P$  is given both as pair of values and a column vector for clarity.

$$P = (4, 3) \quad P = \begin{vmatrix} 4 \\ 3 \end{vmatrix}$$

$$P = \alpha \begin{vmatrix} 1 \\ 0 \end{vmatrix} + \beta \begin{vmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{vmatrix}$$

1. Impossible
2. alpha=3, beta=4
3. alpha=1, beta=3\*sqrt(2)
4. alpha = sqrt(3), beta = sqrt(4)
5. alpha = sqrt(3), beta=sqrt(3)

Question 11: 2D Rotation without Trigonometry (9 Points)

In the figure the mathematics associated with a 2D rotation  $R$  is developed. You need to fill in the correct values for the elements in the rotation matrix, specifically,

$$A = \begin{vmatrix} 3 \\ 4 \end{vmatrix} \quad B = \begin{vmatrix} 9 \\ 12 \end{vmatrix} \quad \begin{vmatrix} 5 \\ 0 \end{vmatrix} = RA \quad \begin{vmatrix} 15 \\ 0 \end{vmatrix} = RB$$

$$R = \begin{vmatrix} a/c & b/c \\ -b/c & a/c \end{vmatrix} \quad \text{Below specify values for } a, b \text{ and } c.$$

$$a=3$$

$$b=4$$

$$c=5$$

Question 12: Perspective Projection Basics (7 Points)

The equation shown in the attached figure illustrates a 3-D point being projected to an image plane at  $z=0$ . What must  $d$  equal for the projection to operate as shown?

$$\begin{pmatrix} 5 \\ 5 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{pmatrix} \begin{pmatrix} 10 \\ 10 \\ 10 \\ 1 \end{pmatrix}$$

$d = 10$

Question 13: 3-D to 2-D Projection (5 Points)

What option below best describes the type of 3-D to 2-D projection accomplished by the matrix shown in the attached figure as the parameter  $d$  approaches infinity.

$$M = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{vmatrix}$$

1. Affine
2. Inverse
3. Orthographic
4. Perspective
5. Homogeneous
6. Vector

Question 14: Camera Pointing (9 Points)

A camera's View Reference Point (VRP) is at (11, 15, 20) and the LookAt point (L) is at (10, 11, 12). The process of building the rotation matrix to orient the camera begins by numerically solving for the three components of the associated View Plane Normal (VPN). Fill in below VPN components. Please express your answers as fractions written precisely in the form integer/integer, in other words in the form  $3/2$ .

$VPN_x$	1/9
$VPN_y$	4/9
$VPN_z$	8/9

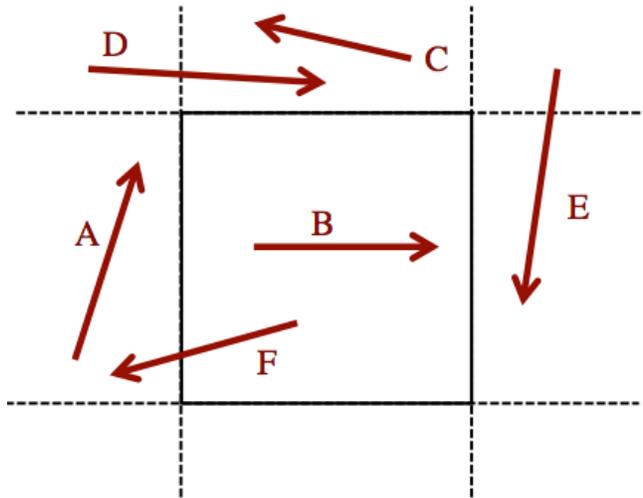
Question 15: Camera Pointing Steps (5 Points)

Here are six steps associated with determining the 3-D to 3-D transformation used to place points in a world or scene model into a camera's reference frame. Indicate the correct order by writing step numbers, 1 to 6, in the left column of the table.

Step #	Operation
<b>6</b>	Compose the rotation and translation into a single 4x4 homogeneous transformation matrix $M$ taking vertices from scene to camera coordinates.
<b>5</b>	Take the cross product of the view plane normal and the horizontal axis basis vector in order to find the basis vector for the camera's vertical axis.
<b>1</b>	Translate so that the origin is placed at the camera View Reference Point.
<b>2</b>	Determine the z axis of the camera expressed in world coordinates, in other words the view plane normal.
<b>3</b>	Take the cross product of the up vector and the view plane normal in order to determine the direction of the basis vector associated with the camera's horizontal axis.
<b>4</b>	Normalize the vector indicating the direction of the camera's horizontal axis and load these values into the top row of rotation matrix.

Question 16: Cohen-Sutherland Clipping (6 Points)

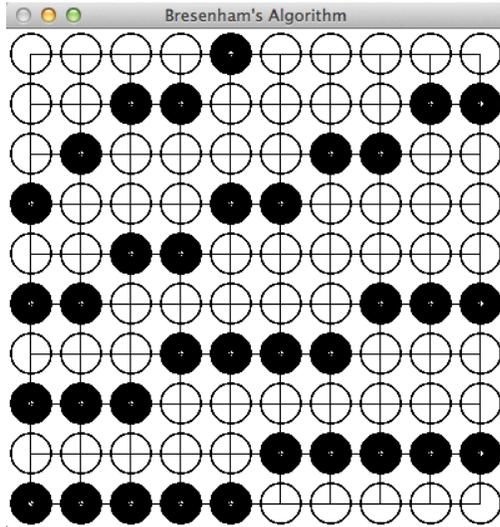
Cohen-Sutherland is based upon assigning a four bit encoding (top, right, bottom, left) to the first and second endpoints of a line segment. Then the rules for clipping are based upon first taking the AND of the bit strings, then the OR of the bit strings, and finally clipping when needed. In the attached figures you are shown six directed line segments (arrow points from first to second endpoint). Match the line segments to the paired four bit encodings below and write the proper labels, A through F, on the proper row of the table.



Line	P1	P2
D	1001	1000
F	0000	0001
B	0000	0000
C	1000	1000
A	0001	0001
E	1100	0100

Question 17: Bresenham's Algorithm (5 Points)

In the figure below four lines have been scan converted using Bresenham's Algorithm. The lines begin on the left side of the figure at pixels (0,0), (0,2), (0,4) and (0,6) respectively. Below match the sequence of decision variable values to the corresponding line. Indicate matches with arrow from left column across to matching cell in the right column.



Line	Decision Value Sequence
Line starting at pixel(0,0)	-5 -1 3 -11 -7 -3 1 -13 -9
Line starting at pixel(0,2)	2 0 -2 4
Line starting at pixel(0,4)	-7 -5 -3 -1 1 -15 -13 -11 -9
Line starting at pixel(0,6)	-1 7 -3 5 -5 3 -7 1 -9

Question 18: Line Representation Conversion 1 (3 Points)

Consider the  $y = m x + b$  (slope-intercept) form of the line from (1,2) to (5,14). What is the slope  $m$ ?

$$m = 3$$

Question 19: Line Representation Conversion 2 (4 Points)

Consider the implicit form of a line  $F(x,y) = ax + by + c = 0$  for the line from  $(1,2)$  to  $(7,10)$ .

If  $a = -4$  then the value of  $c$  must be?

$$c = -2$$

Question 20: Equation of a 3-D Plane (6 Points)

Consider a plane defined by the normal vector  $(2,3,6)$  and the resulting implicit form  $F(x,y,z) = \frac{2}{7}x + \frac{3}{7}y + \frac{6}{7}z + d = 0$

The plane includes the point  $(5,5,4)$ . What is the value  $d$ ?

$$d = -7$$