Lecture 8: Intersecting Parametric Forms

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A1 = [1, 2]; A2 = [6, 1]
B1 = [2, 6]; B2 = [5, 1]
LA = line([A1, A2], color="darkred")
LB = line([B1, B2], color="darkgreen")
bnd = 7
show(LA + LB, xmin=0, ymin=0, xmax=bnd, ymax=bnd, aspect_ratio=1)
Surface the Parametric Form

\[ \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} a_1 x \\ a_1 y \end{pmatrix} + \left( \begin{pmatrix} a x^2 \\ a y^2 \end{pmatrix} - \begin{pmatrix} a_1 x \\ a_1 y \end{pmatrix} \right) t \]

SageMath draws an identical figure.
Lines Know No Bounds
Any Value of $t$ Will Do

• Consider how to draw un-bounded line
• Not really possible
  – This is suggesting graphics concept of clipping
• But for the illustration large bounds work
  – So notice $t$-values are between -10 and 10
Sample Parametric Line
Code to Sample

```python
k = 32.0
Alv = vector(A1); A2v = vector(A2);
B1v = vector(B1); B2v = vector(B2);
tss = [i/(k-1) for i in range(k)]
pts = [Alv + (A2v - Alv) * t for t in tss]
pts = [B1v + (B2v - B1v) * t for t in tss]
gpts = [point(p,color='darkred') for p in pts]
gptsb = [point(p,color='darkgreen') for p in ptsb]
bn = 7
show(sum(gpts) + sum(gptsb), xmin=0, ymin=0, xmax=bn, ymax=bn, aspect_ratio=1)
```

- Python is making this ‘easy’
- How, by allowing enumeration of list elements
- Here 32 points are created that are evenly sampled along the two line segments
How About Intersection

• Pair of equations in two unknowns

\[-(ax_1 - ax_2)t + ax_1 = -(bx_1 - bx_2)s + bx_1\]

\[-(ay_1 - ay_2)t + ay_1 = -(by_1 - by_2)s + by_1\]

• We will shortly also consider turning this into a matrix inversion problem, but not yet.
Solution

• Courtesy of SageMath solve command

\[
s = -\frac{a x_2 (ay_1 - by_1) - a x_1 (ay_2 - by_1) - (ay_1 - ay_2) b x_1}{(ay_1 - ay_2) b x_1 - (ay_1 - ay_2) b x_2 - a x_1 (by_1 - by_2) + a x_2 (by_1 - by_2)}
\]
\[
t = \frac{(ay_1 - by_2) b x_1 - (ay_1 - by_1) b x_2 - a x_1 (by_1 - by_2)}{(ay_1 - ay_2) b x_1 - (ay_1 - ay_2) b x_2 - a x_1 (by_1 - by_2) + a x_2 (by_1 - by_2)}
\]

• For our specific example

\[
\begin{bmatrix}
  s &= \left(\frac{21}{22}\right), \\
  t &= \left(\frac{17}{22}\right)
\end{bmatrix}
\]
Now Draw It

```python
ax1 = A1[0]; ax2 = A2[0]; ay1 = A1[1]; ay2 = A2[1]
bx1 = B1[0]; bx2 = B2[0]; by1 = B1[1]; by2 = B2[1]
eq1v = ax1 + (ax2 - ax1) * t == bx1 + (bx2 - bx1) * s
eq2v = ay1 + (ay2 - ay1) * t == by1 + (by2 - by1) * s
resv = solve([eq1v, eq2v], s, t)

tstar = resv[0][1].rhs()
LA = line([A1, A2], color="darkred")
LB = line([B1, B2], color="darkgreen")
poi = point((lapx(tstar), lapy(tstar)), size=64, color='orange')
bn = 7
show(LA + LB + poi, xmin=0, ymin=0, xmax=bn, ymax=bn, aspect_ratio=1)
```