Lecture 12: Illumination
CS 410

September 27, 2018
Goals of Reflectance

• Model simple light sources
  – Point light sources
  – Extended light sources
  – Ambient lighting

• Model lighting geometry
  – Angle between light ray and surface normal
  – Angle between surface normal and viewing ray

• Model surface reflections
  – Diffuse (Lambertian) reflections
  – Specular reflections
  – Phong reflectance
Reflectance - 3 Kinds

• Ambient reflection.
  – Models perfectly even illumination.

• Lambertian reflection.
  – Matte surfaces, dull rather than shiny.

• Specular reflection.
  – Bright highlights seen on polished surfaces.
Reflectance - Geometry

Reflectance functions are defined in terms of:

- The position of a (point) light source
- The orientation of the reflecting surface
- The viewing position.

![Diagram showing the relationship between light source, surface normal, viewing angle, and angle of incidence.]

CSU CS 410 2018 ©Ross Beveridge & Bruce Draper
Reflectance Functions (III)

• Light rays, surface normals, and colors are expressed using vectors
  – Light often described as illumination \( I \)
  – Three components to illumination:

\[
I = \begin{bmatrix}
  i_r \\
  i_g \\
  i_b
\end{bmatrix}
\]
Lambertion Reflection

• Consider a point light source and a Diffuse (Lambertian) Surface.

• Diffuse surfaces reflect light with equal intensity in all directions.
Lambertian Reflection.

The moon is cited as a good example of Lambertian reflection

Maybe not such a great example ... large dark patches .... But you can tell where the sun is ...
A Touch of Physics

Light per unit area arriving depends upon angle to light source.

Light leaves in all directions, reflected off micro surfaces.
Diffuse Reflection

• Illumination/area drops by $\theta$

$$i_r = b_r \cos \theta \quad i_g = b_g \cos \theta \quad i_b = b_b \cos \theta$$

• Where light L emits light with brightness B:

$$B = \begin{bmatrix} b_r \\ b_g \\ b_b \end{bmatrix}$$
Diffuse Reflection – Parameters

• Diffuse parameters modulate brightness

\[
I = \begin{bmatrix}
  i_r \\
  i_g \\
  i_b
\end{bmatrix}
= \begin{bmatrix}
  k_r & 0 & 0 \\
  0 & k_g & 0 \\
  0 & 0 & k_b
\end{bmatrix}
\begin{bmatrix}
  b_r \\
  b_g \\
  b_b
\end{bmatrix}
\cos(\theta)
\]

• The same statement, but more concise:

\[
I_d = K_d B \cos(\theta)
\]

The subscript \(d\) added to indicate diffuse illumination
Rare Case Still Worth Mention

- Use the full power of the 3x3 matrix
  - Comes in red, goes out green

\[
I = \begin{pmatrix}
i_r \\ i_g \\ i_b
\end{pmatrix} = \begin{pmatrix}
k_{rr} & k_{rg} & k_{rb} \\ k_{gr} & k_{gg} & k_{gb} \\ k_{br} & k_{bg} & k_{bb}
\end{pmatrix} \begin{pmatrix}
b_r \\ b_g \\ b_b
\end{pmatrix} \cdot \cos(\theta)
\]
Example of Non-Diagonal Reflectance Matrix

- Oil slicks reflect light of one color as another color
- Thin-film interference effect
- Given angles, can be modeled as a non-diagonal $K_d$ matrix

http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/oilfilm.html
Material Properties (II)

• There are some pragmatic constraints:
  – No term in K may exceed 1.0
  – No term in K may be negative
  – No column may have a sum exceeding 1

• But, …

• These constraints are not always enforced

There is an art to lighting a scene!
Diffuse Reflection – Computation

• The final term in diffuse reflection is \( \cos(\theta) \).
  – Defined by angle between the incoming light ray \( L \) and the surface normal \( N \)

\[
L = \frac{Q - S}{|Q - S|} \\
N = \frac{E_1 \times E_2}{|E_1 \times E_2|} \\
cos(\Phi) = N \cdot L
\]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Unit Length Surface Normal</td>
</tr>
<tr>
<td>L</td>
<td>Unit Length Vector to Light</td>
</tr>
<tr>
<td>Q</td>
<td>3D Position of the Light</td>
</tr>
<tr>
<td>S</td>
<td>3D Position on Surface</td>
</tr>
<tr>
<td>P1, P2 &amp; P3</td>
<td>3D Vertices of the Triangle</td>
</tr>
<tr>
<td>E1 &amp; E2</td>
<td>Two 3D Edges of the Triangle</td>
</tr>
</tbody>
</table>
Surface Normals - Sphere

- What about the surface normal for a sphere?

\[ N = \frac{S - P_C}{\|S - P_C\|} \]

Note: the normal to a polygon is constant across the polygon; for a sphere it is a function of the surface point.
Many Light Sources

• For a single point light source:

\[ I = K_d B (N \cdot L) \]

• For m point light sources

\[ I = \sum_{i=1}^{m} K_d B_i (N \cdot L_i) \]
Ambient Reflection

- Ambient reflection is just diffuse reflection of light from the “ambient” light source
- Ambient light models background light
  - It is the same intensity everywhere
  - It is the same color everywhere
  - It comes from every angle
- So…

\[ I_a = K_d B_a \]
Specular Reflection

• Most surfaces do more.
• Consider two extremes:
  – 1: a diffuse surface,
  – 2: a mirror.
• Specular reflection component
  – Mirrors are extreme
  – Mirrors reflect about a single angle
• Metals, for example, combine
  – Diffuse and specular components.
Only Skin Deep

• Diffuse models light “deep” reflection
  – Enters through micro-holes in the surface
  – Bounced from facet to facet,
    • Possibly changing color
  – Exits at a random angle

• Specular reflection is “surface” reflection
  – Light hits a single surface facet, skips off.
  – The color of the light is unchanged.
  – The exit angle depends on the entry angle.
    • Catchy Phrase – equal and opposite.
Limiting Case - Mirror

- Angle of incidence = Angle of reflection.
- Idealized form only applies to mirrors.
Reflection – Mirror Like Surface
Light Source Reflecting
Reflection and Viewer

• V is the unit vector from the surface point to the camera:
  \[ V = \frac{(C - S)}{|C - S|} \]

• Calculating R is more involved
Reflection Vector

• Let NL be the projection of L onto N
  – Assume N is a unit vector
    \[ NL = (L \cdot N)N \]

• T is defined as
  \[ T = NL - L = (L \cdot N)N - L \]

• R is \( L + 2T \)
  \[
  R = L + 2 \left( (L \cdot N)N - L \right) \\
  = 2 (L \cdot N)N - L
  
\]
Back to Perfect (for now)

• If $V \neq R$, there is no (pure) specular reflection.

• If $V = R$, then ….

• Note that $k_s$ is a scalar, measuring the percent of light reflected.
  – Color reflected is that of the light source.

\[ I = k_s B \]

\[ = \begin{bmatrix} b_r \\ b_g \\ b_b \end{bmatrix} \]

Warning: This is the correct mental model, it is not useful in practice.
Close Counts – Phong Reflection

• In Phong’s model, reflection is strongest in the direction of the angle of reflection
• Drops off with the cosine of the deviation from the angle of reflection
Phong Specular Highlights

\[ I = k_s B (\cos \Phi)^\alpha \]

- \( \Phi \) is the angle between the viewing ray and the angle of reflection.
- \( \alpha \) is the so-called Phong constant, expressing how “shiny” an object is
  - Mirrors: \( \alpha = 200 \)
  - Dull objects: \( \alpha = 5 \ldots 50 \)
Appearance with Changing $\alpha$

• Consider which has the larger alpha?
Phong - The Algebra

- \( \cos(\Phi) = V \cdot R = V \cdot (2(L \cdot N)N - L) \)

\[
I = k_s B (V \cdot R)^\alpha
\]

\[
I = k_s B \left( V \cdot (2(L \cdot N)N - L) \right)^\alpha
\]
Total Reflectance

- Reflectance off a surface point is the sum of:
- Reflection from the ambient light
- Diffuse reflection off of every point light
- Specular reflection off of every point light

\[ I = K_a B_a + \sum_{i \in \text{lights}} \left( K_d B_i (L_i \cdot N) + k_s B_i (V \cdot R_p)^\alpha \right) \]

\[ R = 2(L \cdot N)N - L \]
No ‘Negative’ illumination

- The influence of a light source cannot be negative
  - They are light *emitters*
- We assume $N \cdot V \geq 0$
  - Where $V$ points from surface toward $fp$
  - Otherwise, flip $N$
- Therefore, if $L_{ip} \cdot N < 0$, set it to 0
- Similarly, if $V \cdot R_p < 0$, set it to 0
- These situations occur when the light source is behind the surface
  - The light source is shining on the other side of the (opaque) surface