Uncertainty

Lecture #16

10/23/08

Stepping Back…

- We have finished talking about logic
  - 2nd Programming assignment is due a week from Tuesday
  - Now would be a good time to ask questions
- We have also finished talking about search
  - Although search is a component to everything in this class
- Our next topic is uncertainty…
  - Chapter 13 of your text (today)
  - Chapter 14 of your text (Tuesday)

Uncertainty

Is every proposition true or false?

Sources of Uncertainty

- Consider the following statements:

1. It rained in Tuscaloosa last Friday

   Uncertainty caused by ignorance
   Refers to a state of knowledge

2. It will rain in Ft. Collins tomorrow

   Predictive uncertainty
   Refers to a sampling likelihood

3. The odds of global nuclear war is 1 in 1 million

   Subjective estimate
   Subjective uncertainty

The Language of Probability

- Random variables
  - The equivalent of a proposition in logic
  - Each variable has a set of possible values
    - Boolean
    - Discrete (finite list of possibilities)
    - Continuous

LoP (cont.)

- An atomic event is the *complete* specification of the state of world
  - The world may contain many random variables
  - Alternatively, it may contain only one
  - An atomic event specifies the value of every random variable
- Rain example:
  - It DID rain in Tuscaloosa and WON'T rain in Ft. Collins.
- We can assign probabilities to atomic events
  - What is the probability that is rained in Tuscaloosa but not Ft. Collins?

Unconditional Probabilities

- The *unconditional* or *prior* probability of an atomic event is our state of belief in the event prior to any specific information
  - P(rain in Tuscaloosa) = .3
    It rains a lot in Alabama….
  - P(rain in Ft. Collins) = .1
    Not so much in Colorado…

Conditional Probabilities

- The conditional probability of an atomic event is its probability, *given that the values of some of the random variable are known.*
- Example:
  (P(Ft. Collins = rain | Tuscaloosa = rain) = ?

Bayes Rule

$$P(a \mid b) = \frac{P(a \wedge b)}{P(b)}$$

Bayes Rule (II)

- The previous equation holds for P(b) > 0
  - What happens when P(b) = 0?
- Bayes rule can be rewritten as the product rule:

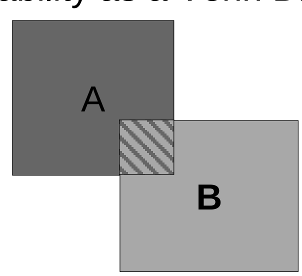
$$P(a \wedge b) = P(a \mid b)P(b)$$

The Axioms of Probability

- All of probability theory can be derived from 3 axioms:
  - Axiom 1: $\forall a,\ 0 \le P(a) \le 1$
  - Axiom 2: P(true) = 1, P(false) = 0
  - Axiom 3: P(a v b) = P(a) + P(b) – P(a^b)
- Derivable from above:
  - Lemma 1: $\Sigma P(a) = 1$
    - Summed over all possible values of a

What does P(a) = .3 actually mean?

Probability as a Venn Diagram



P(a v b) = P(a) + P(b) – P(a^b)

Joint Distributions

- Let T be the proposition of rain in Tuscaloosa
- Let C be the propositiion of rain in Ft. Collins

	C	¬C
T	.03	.27
¬T	.07	.63

Joint Distributions (II)

- Book example: probabilities of (1) having a cavity, (2) having a toothache, and (3) having the dentist's probe catch.

	toothache		¬toothache	
	catch	¬catch	catch	¬catch
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

Marginal Probabilities

- The marginal probability is the probability of one term, summed over the others
  - The probability of cavity is 0.2. Why?
  - The probability of toothache is also 0.2. Why?
  - The probability of catch is 0.34. Why?
- As a general rule (marginalization):

$$P(Y) = \sum_z P(Y, z)$$

Conditioning

- Alternatively, you may know conditional probabilities rather than the joint probabilities.
- The conditioning rule is:

$$P(Y) = \sum_z P(Y \mid Z) P(Z)$$

Independence

- Sometimes, two random variables have nothing to do with each other

$$P(a \wedge b) = P(a)P(b)$$

$$P(a \mid b) = P(a)$$

$$P(b \mid a) = P(b)$$

Independence (II)

- Why is independence important?
  - Because its useful to know when two things *aren't* related
  - Because joint probability tables grow exponentially with the number of variables
  - Because sample-based interpretations of too many dependent variables are infeasible

Simple Bayesian Inference

- What is the probability of a cavity, given a toothache and that the dentists probe caught?
  - i.e. what is P(cavity | toothache, catch)?
  - P(toothache ^ catch) = 0.124
  - P(cavity ^ toothache ^ catch) = .108

$$P(a \mid b) = \frac{P(a \wedge b)}{P(b)}$$

$$\frac{.108}{.124} = .871$$