Bayes II
(Introduction to Bayes Nets)

Lecture #17

10/28/08

Announcements

- I received multiple queries about the homework and Section 9.5 in your text
  - It covers FoL resolution theorem proving
  - Like, why didn't I cover it?
- Well, I thought it was implied, but I am happy to cover it if you wish…

FoL Resolution Theorem Proving

- General idea: same as predicate resolution theorem proving
  - (A(x) v B(x)) ^ (¬A(x) v C(x)) ⇒ (B(x) v C(x))
- New issues
  - Universally quantified variables
  - Existentially quantified variables
  - Constants

Conjunctive Normal Form (CNF)

- Same rules as before:
  - Eliminate ⇔ and ⇒
  - Move ¬ to the bottom of the parse tree
  - Make the expression a two level conjunction (and) of disjunctions (ors) of (possibly negated) prepositions
- New rules:
  - Eliminate existential variables (Skolemization)
  - Move universal quantifiers to the top of the tree (where they become implicit)

Conversion to CNF

- Example:

  ∀x [∀y Animal(y) ⇒ Loves(x,y)] ⇒ [∃y Loves(y,x)]

1. Eliminate ⇔

   (nothing to do in this example)
2. Eliminate ⇒

   ∀x [∀y ¬Animal(y) v Loves(x,y)] ⇒ [∃y Loves(y,x)]

   ∀x [¬∀y ¬Animal(y) v Loves(x,y)] v [∃y Loves(y,x)]

Conversion (cont)

3. Move ¬ inwards
   - ¬∀x P(x) ⇒ ∃x ¬P(x)
   - ¬∃x P(x) ⇒ ∀x ¬P(x)

∀x [¬∀y ¬Animal(y) v Loves(x,y)] v [∃y Loves(y,x)]

∀x [∃y ¬[¬Animal(y) v Loves(x,y)]] v [∃y Loves(y,x)]

∀x [∃y ¬¬Animal(y) ^ ¬Loves(x,y)]] v [∃y Loves(y,x)]

∀x [∃y Animal(y) ^ ¬Loves(x,y)]] v [∃y Loves(y,x)]

Conversion (III)

4. Standardize Variables
   - Don’t re-use names across scopes

∀x [∃y Animal(y) ^ ¬Loves(x,y)]] v [∃y Loves(y,x)]
becomes
∀x [∃y Animal(y) ^ ¬Loves(x,y)]] v [∃z Loves(z,x)]

Conversion (IV)

5. Skolemize
   - Replace existential variables by new constants
   - If the existential is inside a universal, it depends on the binding of the universal

∀x [∃y Animal(y) ^ ¬Loves(x,y)]] v [∃z Loves(z,x)]
Becomes
∀x [Animal(F(x)) ^ ¬Loves(x,F(x))]] v [Loves(G(x),x)]

Conversion (V)

6. Drop universal quantifiers
   - They become implicit

   [Animal(F(x)) ^ ¬Loves(x,F(x))]] v [Loves(G(x),x)]
7. Distribute v over ^
   - As in propositional logic

   [Animal(F(x)) v Loves(G(x),x) ^
   [¬Loves(x, F(x)) V Loves(G(x), x)]

Disheartened?

- Don’t be!
  - The initial axioms in the homework assignment never change, so calculate the CNF form by hand
  - The turns of the game only produce two types of axioms
    - (Plum(c) v Revolver(c) v Billiards(c))
    - ¬(Plum(c) v Revolver(c) v Billiards(c))
  - So only automate the conversion of this form!

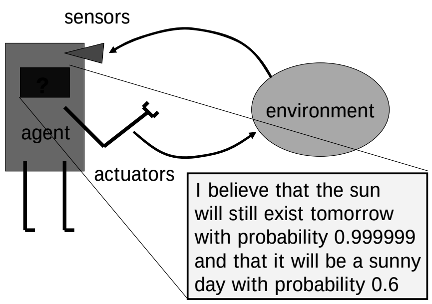
Resolution Theorem Proving

- Now the resolution process is:
  - For any pair of disjunctive clauses that contain P(q) and ¬P(q)
    - Q may be a variable or constant
    - Unify the expressions
      - Variables unify with variables by simple substitution
      - Variables unify with constants by adopting the constant
    - Apply resolution

Announcements (II)

- Programming assignment #2 is due in 1 week
  - Don’t forget to vote!
- No ACM meeting this Wednesday

Probabilistic Agent



Problem

- At a certain time t, the KB of an agent is some collection of beliefs
  – Every belief has a degree of certainty
- At time *t* the agent's sensors make an observation that changes the strength of one of its beliefs
- How should the agent update the strength of its other beliefs?

Purpose of Bayesian Networks

- Facilitate the description of a collection of beliefs by
  – making causality relations explicit
  – exploiting conditional independence
- Provide efficient methods for:
  – Representing a joint probability distribution
  – Updating belief strengths when new evidence is observed

Review

- Probabilities represent a degree of belief
  – Bayesian interpretation
    - Subjective measure of internal confidence
  – Frequentist interpretation
    - Based on sampling from random draws
- Probability values
  – $0 \le p \le 1$
  – P(A v B) = P(A) + P(B) – P(A ^ B)

Review

- Conditional Probability: P(A|B)
  – The probability of A given that B is true
  – The frequency with which A will be observed, considering only samples in which B is true
  – E.g. P(toothache | cavity) *from last lecture*
- Bayes Rule

$$P(a \mid b) = \frac{P(b \mid a)P(a)}{P(b)}$$

Bayesian networks, a.k.a.

- Belief networks
- Causal networks
- Graphical models

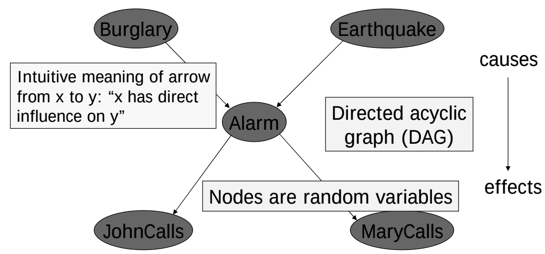
Example

I'm at work, neighbor John calls to say my alarm is ringing, but neighbor Mary doesn't call. Sometime it's set off by a minor earthquake. Is there a burglary?

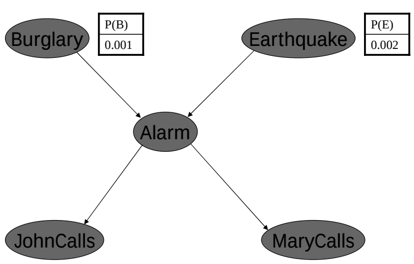
Variables: Burglary, Earthquake, Alarm, JohnCalls, MaryCalls

Network topology reflects "causal" knowledge:
- A burglary can set the alarm off
- An earthquake can set the alarm off
- The alarm can cause Mary to call
- The alarm can cause John to call

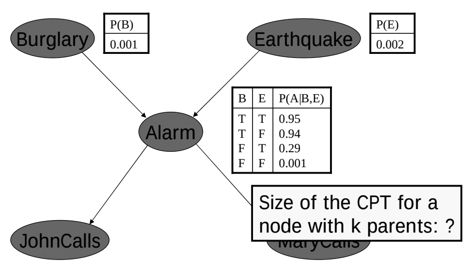
A Simple Network



Assigning Probabilities to Roots



Conditional Probability Tables



Conditional Probability Tables