Propositional Calculus Inference Rules

Antecedent $\vdash$ Consequent	Name
$((p \to q) \wedge p) \vdash q$	Modus Ponens
$((p \to q) \wedge \neg q) \vdash \neg p$	Modus Tollens
$((p \to q) \wedge (q \to r)) \vdash (p \to r)$	Hypothetical Syllogism
$((p \vee q) \wedge \neg p) \vdash q$	Disjunctive Syllogism
$((p \to q) \wedge (r \to s) \wedge (p \vee r)) \vdash (q \vee s)$	Constructive Dilemma
$((p \to q) \wedge (r \to s) \wedge (\neg q \vee \neg s)) \vdash (\neg p \vee \neg r)$	Destructive Dilemma
$(p \wedge q) \vdash p$	Simplification (1)
$(p \wedge q) \vdash q$	Simplification (2)
$p, q \vdash (p \wedge q)$	Conjunction
$p \vdash (p \vee q)$	Addition
$((p \to q) \wedge (p \to r)) \vdash (p \to (q \wedge r))$	Composition
$\neg(p \wedge q) \vdash (\neg p \vee \neg q)$	De Morgan's Theorem (1)
$\neg(p \vee q) \vdash (\neg p \wedge \neg q)$	De Morgan's Theorem (2)
$(p \vee q) \vdash (q \vee p)$	Commutation (1)
$(p \wedge q) \vdash (q \wedge p)$	Commutation (2)
$(p \vee (q \vee r)) \vdash ((p \vee q) \vee r)$	Association (1)
$(p \wedge (q \wedge r)) \vdash ((p \wedge q) \wedge r)$	Association (2)
$(p \wedge (q \vee r)) \vdash ((p \wedge q) \vee (p \wedge r))$	Distribution (1)
$(p \vee (q \wedge r)) \vdash ((p \vee q) \wedge (p \vee r))$	Distribution (2)
$p \vdash \neg \neg p$	Double Negation (1)
$\neg \neg p \vdash p$	Double Negation (2)
$(p \to q) \vdash (\neg q \to \neg p)$	Transposition
$(p \to q) \vdash (\neg p \vee q)$	Material Implication
$(p \leftrightarrow q) \vdash ((p \to q) \wedge (q \to p))$	Material Equivalence (1)
$(p \leftrightarrow q) \vdash ((p \wedge q) \vee (\neg q \wedge \neg p))$	Material Equivalence (2)
$(p \leftrightarrow q) \vdash ((p \vee \neg q) \wedge (q \vee \neg p))$	Material Equivalence (3)
$((p \wedge q) \to r) \vdash (p \to (q \to r))$	Exportation
$(p \to (q \to r)) \vdash ((p \wedge q) \to r)$	Importation
$p \vdash (p \vee p)$	Tautology (1)
$p \vdash (p \wedge p)$	Tautology (2)
$\vdash (p \vee \neg p)$	Tertium non datur
$(p \vee q), \neg p \vdash q$	Resolution

You may rip this page off; you do not need to turn it in

(Don’t write on it, however, if you want partial credit for your work)

Midterm Exam

Oct. 11, 2007

Introduction to Artificial Intelligence
(CS440)

Your Name: ____

Note: this is a challenging test. Do not panic. Answer as many questions as you can as well as you can. Grades will be assigned on a curve.

Two hints:

1. Many questions are multi-part. For full credit, answer *all* parts of the question.
2. Show your work. I can't give you partial credit if I don't know how you arrived at your answer.

Do not write on this page
(for grading purposes only)

Question	Max Points	Earned
1	25	
2	25	
3	10	
4	10	
5	5	
6	10	
7	5	
8	5	
9	5	
Total	100	

Question #1 (25 points):

Using traditional forward chaining with named inference rules, prove ¬A given the following axioms: A ⇒C, B ⇔ C, ¬(B ^ D), D. You may use the space below, and any inference rule in the table provided.

Antecedent	Inference Rule	Consequent

Question #2 (25 points):

1) Prove ¬A from the same axioms as in question #1, this time using resolution. For partial credit, be sure to show me all the steps by which you (a) converted the axioms into CNF form and (b) every application of the resolution operator.

Question 3 (short answer; 10 points)

Translate the following natural language paragraph into predicate (first-order) logic, (You do <u>not</u> have to prove it – in fact, you can't – just state the axioms and the assertion to be proved.)

"In order to graduate from the CS department, one must take at least 4 400-level courses. CS410, CS414, CS420, CS430, CS440, CS451, CS457, and CS475 are all 400-level courses. Sarah has graduated from the CS department. Prove Sarah has taken CS440."

Questions #4-7

Consider propositional (zeroth-order) resolution theorem proving as an uninformed search problem. In particular, assume you are handed an expression in CNF, and the expression has M conjunctions, each containing N terms, with a total of L unique literals (i.e. there are L propositions).

Question #4 (10 pts): Formalize resolution theorem proving as an uninformed search problem. Don't forget to define the states, the initial state, the successor function, the goal test and the path cost.

Question #5 (5 pts): What is the nature of the resolution theorem proving search space? Is it a tree, a directed graph, an undirected graph, or something else? Explain why.

Question #6 (10 pts): What is the complexity of uninformed search over this space. In particular, what are the maximum branching factor, the minimum solution depth, and the maximum search depth?

Question #7 (5 pts): Give an admissible heuristic for propositional (zeroth-order) resolution as a search problem. Explain why it is admissible.

Questions 8-9: It is possible to prove theorems, not by resolution or uninformed forward search, but by *constraint propagation*. In particular, you can search for a model (i.e. a binding of every literal to either T or ⊥) , viewing every axiom as a constraint. With this in mind, answer the following questions using the following set of axioms:

$$(A \vee B), (C \Rightarrow \neg A), C$$

Question #8 (5 pts): Are there any unary constraints? If so, what are they?
Are there any binary constraints? If so, what are they?
Are there any ternary constraints? If so, what are they?

Question #9 (5 pts): List the remaining domains for A, B and C after the initial constraint propagation has taken place, but before any search has begun. Explain how you arrived at your answer.