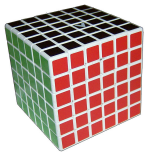
Problem Solving by Searching

Russell and Norvig, chapter 3.1-3.3

Puzzles!

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9



The missionaries and cannibals problem

- Goal: transport the missionaries and cannibals to the right bank of the river.
- Constraints:
  - Whenever cannibals outnumber missionaries, the missionaries get eaten
  - Boat can hold two people and can't travel empty

Formulating the problem

- A state description that allows us to describe our state and goal:

$$(M_L, C_L, B)$$

M_L: number of missionaries on left bank
C_L: number of cannibals on left bank
B: location of boat (L,R)

- Initial state: (3,3,L) Goal: (0,0,R)

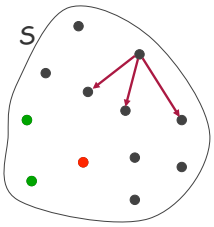
Problem solving agents

- Problem Formulation
  - States and actions (successor function).
- Goal Formulation
  - Desired state of the world.
- Search
  - Determine a sequence of actions that lead to a goal state.
- Execute
  - Perform the actions.
- Assumptions:
  - Environment is fully observable and deterministic
  - Agent knows the effects of its actions

Graph formulation of the problem

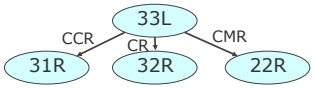
- Nodes: all possible states.
- Edges: edge **from** state *u* to state *v* if *v* is reachable from *u* (by an action of the agent)
- Edges for missionaries and cannibals problem?
- Problem is now to find a path from (3,3,L) to (0,0,R).
- In general, paths will have costs associated with them, so the problem will be to find the lowest cost path from initial state to the goal.

Formulating a search problem



- State space S (nodes)
- Successor function: the states you can move to by an action (edge) from the current state
- Initial state
- Goal test
  is a state x a goal?
- Cost

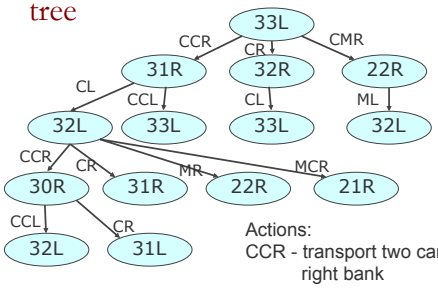
Back to our problem



State: 33L three missionaries and three cannibals on left bank, boat is on left bank
Actions (operators):
CCR - transport two cannibals to the right bank
MCL - transport a missionary and a cannibal to the left bank

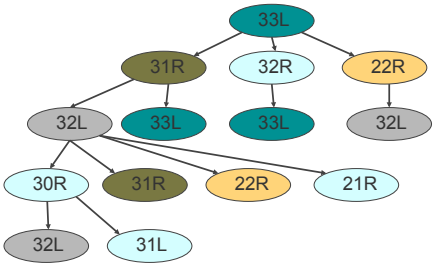
Why no MMR transition from this state?

The (partially expanded) search tree



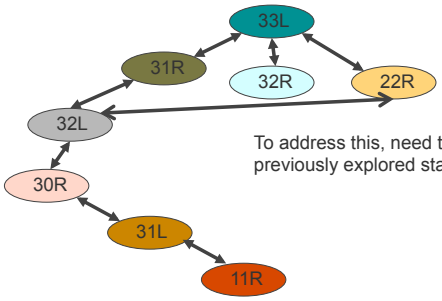
Actions:
CCR - transport two cannibals to the right bank
MCL - transport a missionary and a cannibal to the left bank

Repeated states



The search tree contains duplicate nodes, since the underlying graph in which we are searching is not a tree!

The state space graph



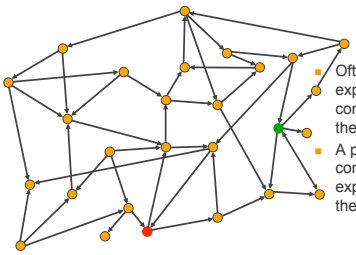
To address this, need to track previously explored states

Aside: Searching the graph

function GRAPH-SEARCH(*problem*) returns a solution, or failure
  initialize the *frontier* using the initial state of problem
  initialize the *explored* set to be empty
  loop do
    if the *frontier* is empty **then return** failure
    choose a leaf *node* and remove it from the frontier
    if the *node* contains a goal state **then return** the corresponding solution
    add the *node* to the *explored* set
    expand the chosen *node*
    add the resulting *nodes* to the *frontier* only if not in the *frontier* or *explored*

The difference between BFS and DFS is in how the node is chosen and/or how it is added to the frontier: queue versus stack.

Searching the state space



- Often it is not feasible (or too expensive) to build a complete representation of the state graph
- A problem solver must construct a solution by exploring a small portion of the graph

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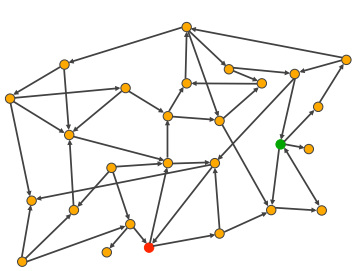
Searching the state space



Search tree

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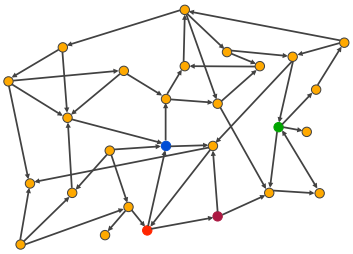
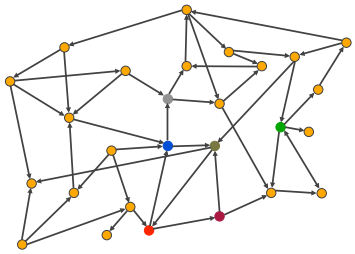
Searching the state space



Search tree

15

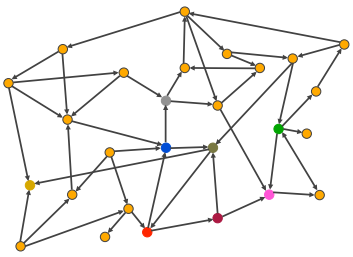
Searching the state space



Search tree

16

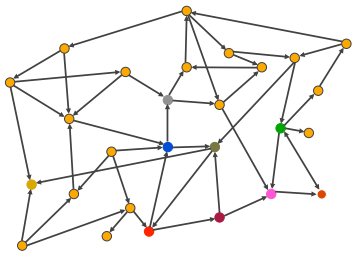
Searching the state space



Search tree

17

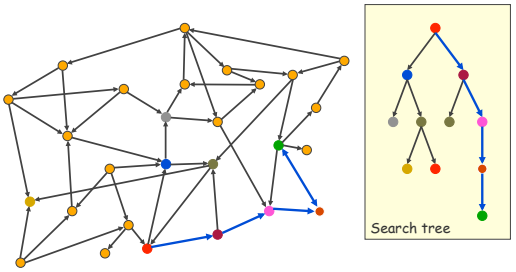
Searching the state space



Search tree

18

Searching the state space



The 8 puzzle

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

- States?
- Initial state?
- Actions?
- Goal test?
- Path cost?

The 8 puzzle

7	2	4
5		6
8	3	1

Start State

	1	2
3	4	5
6	7	8

Goal State

- States? Integer location of each tile. How many of them are there?
- Initial state? Any state
- Actions? (tile, direction)
  where direction is one of {*Left*, *Right*, *Up*, *Down*}
- Goal test? Check whether goal configuration is reached
- Path cost? Number of actions to reach goal
- Is the search graph a tree?

(n^2-1)-puzzle

8	2	
3	4	7
5	1	6

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

. . . .

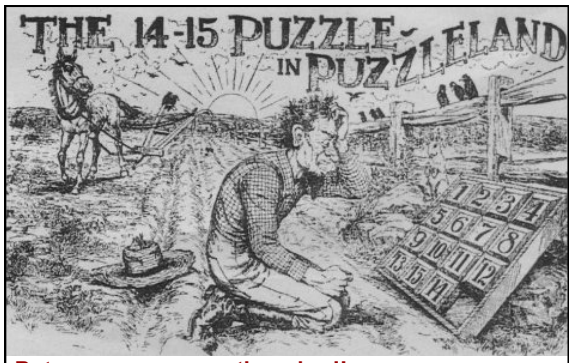
The 15-puzzle

Sam Loyd offered $1,000 of his own money to the first person who would solve the following problem:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

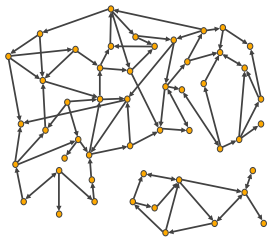
?

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	



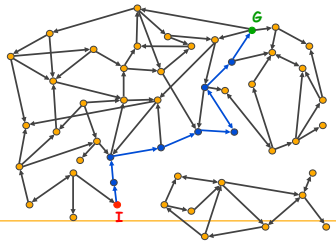
But no one ever won the prize !!

Why?



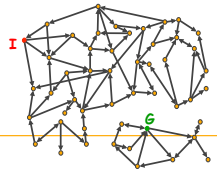
Solution to the Search Problem

- A solution is a path connecting the initial node to a goal node (any one)



Solution to the Search Problem

- A solution is a path connecting the initial node to a goal node (any one)
- The cost of a path is the sum of the edge costs along this path
- An optimal solution is a solution path of minimum cost
- There might be no solution !



Path Cost

- An edge cost is a positive number measuring the "cost" of performing the action corresponding to the edge, e.g.:
  - 1 in the 8-puzzle example
- We will assume that for any given problem the cost c of an edge always satisfies:

  $c \geq \varepsilon > 0$, where ε is a constant
  - Why? Has to do with the cost of arbitrarily long paths

Goal State

- It may be explicitly described:

1	2	3
4	5	6
7	8	

- or partially described:

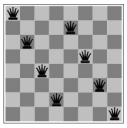
1	a	a
a	5	a
a	8	a

("a" stands for "any" other than 1, 5, and 8)

- or defined by a condition, e.g., the sum of every row, of every column, and of every diagonal equals 30

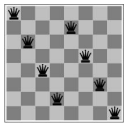
15	1	2	12
4	10	9	7
8	6	5	11
3	13	14	

Another example: the 8 queens problem



- Incremental vs. complete state formulation:
  - Incremental formulation starts with an empty state and involves operators that augment the state description
  - A complete state formulation starts with all 8 queens on the board and moves them around

8 queens problem: representation is key



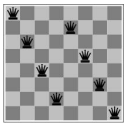
Incremental formulation

- States? Any arrangement of 0 to 8 queens on the board
- Initial state? No queens
- Actions? Add queen in empty square
- Goal test? 8 queens on board and none attacked
- Path cost? None

  64 x 63 x … 57 ~ 3 x 10^{14} states to investigate

Is the search graph a tree?

A better representation

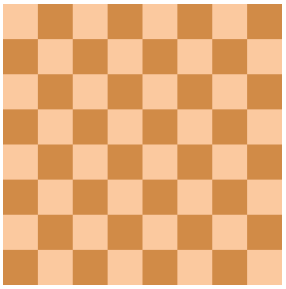


Another incremental formulation:

- States? n (between 0 and 8) queens on the board, one in each of the n left-most columns; no queens attacking each other.
- Initial state? No queens
- Actions? Add queen in leftmost empty column such that it does not attack any of the queens already on the board.
- Goal test? 8 queens on board

2057 possible sequences to investigate

8 queens solved

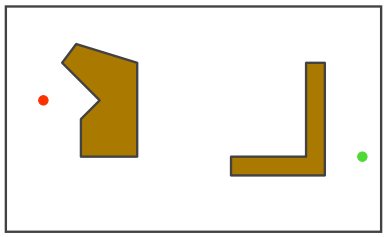


Animated gif from https://sites.google.com/a/lclark.edu/drake/courses/ai/search-and-n-queens

n-queens problem

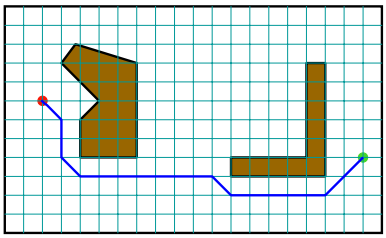
- A solution is a goal node, not a path to this node (typical of design problem)
- Number of states in state space:
  - 8-queens → 2,057
  - 100-queens → 10^{52}
- But techniques exist to solve n-queens problems efficiently for large values of n

Path planning



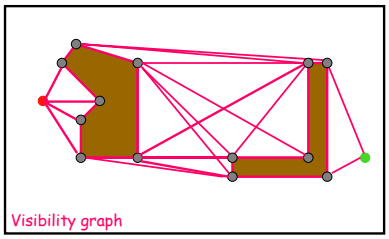
What is the state space?

Path planning



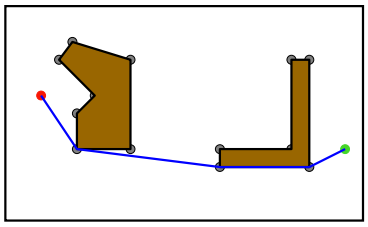
This path is the shortest one in the discretized space, but not in the original space.

An alternative formulation



Cost of a step is the length of the step

An alternative formulation



The solution in this space is the same as in the continuous space!

Lozano-Pérez, Tomás; Wesley, Michael A. (1979), "An algorithm for planning collision-free paths among polyhedral obstacles", *Communications of the ACM* **22** (10): 560–570,

Assumptions in Basic Search

- The world is static
- The world is discretizable
- The world is fully observable
- The actions are deterministic

Search remains an important tool even if not all these assumptions are satisfied

Search and AI

- Search methods are ubiquitous in AI systems.
- An autonomous robot uses search to decide which actions to take and which sensing operations to perform, to quickly anticipate collision, to plan trajectories,…
- Logistics companies use search to allocate resources, plan routes, schedule employees…
- Agents or opponents in games use search to navigate, select moves, anticipate other's actions, and decide on their own actions.

Applications

- Search plays a key role in many applications, e.g.:
  - Route finding (google/mapquest, internet, airline)
  - VLSI Layout
  - Robot navigation.
  - Pharmaceutical drug design, protein design
  - Video games