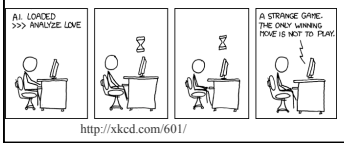
Adversarial Search and Game Playing

Russell and Norvig, Chapter 5



http://xkcd.com/601/

Games

- Games: multi-agent environment
  - What do other agents do and how do they affect our success?
  - Cooperative vs. competitive multi-agent environments.
  - Competitive multi-agent environments give rise to adversarial search a.k.a. games
- Why study games?
  - Fun!
  - They are hard
  - Easy to represent and agents restricted to small number of actions… sometimes!

Relation of Games to Search

- Search – no adversary
  - Solution is (heuristic) method for finding goal
  - Heuristics and CSP techniques can find optimal solution
  - Evaluation function: estimate of cost from start to goal through given node
  - Examples: path planning, scheduling activities
- Games – adversary
  - Solution is strategy (strategy specifies move for every possible opponent reply).
  - Time limits force approximate solutions
  - Examples: chess, checkers, Othello, backgammon

Types of Games

	Deterministic	Chance
Perfect information	chess, go, checkers, othello	backgammon
Imperfect information	Bridge, hearts	Poker, canasta, scrabble

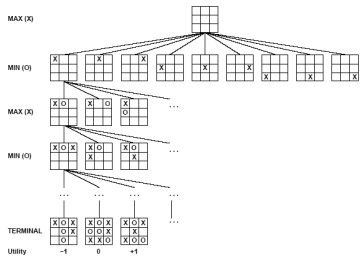
Our focus: deterministic, turn-taking, two-player, zero-sum games of perfect information

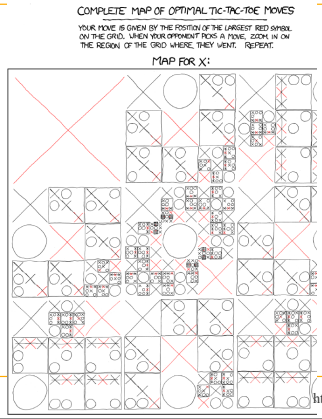
zero-sum game: a participant's gain (or loss) is exactly balanced by the losses (or gains) of the other participant.
perfect information: fully observable

Game setup

- Two players: MAX and MIN
- MAX moves first and they take turns until the game is over.
- Games as search:
  - Initial state: e.g. starting board configuration
  - Player(s): which player has the move in a state
  - Action(s): set of legal moves in a state
  - Result(s, a): the states resulting from a given move.
  - Terminal-test(s): game over? (terminal states)
  - Utility(s,p): value of terminal states, e.g., win (+1), lose (-1) and draw (0) in chess.
- Players use search tree to determine next move.

Partial Game Tree for Tic-Tac-Toe





http://xkcd.com/832/

The Tic-Tac-Toe Search Space

- Is this search space a tree or graph?
- What is the minimum search depth?
- What is the maximum search depth?
- What is the branching factor?

Optimal strategies

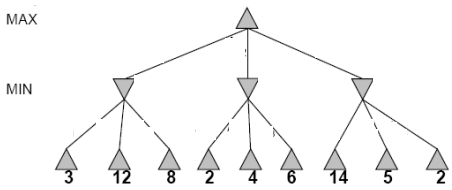
- Find the best *strategy* for MAX assuming an infallible MIN opponent.
- Assumption: Both players play optimally.
- Given a game tree, the optimal strategy can be determined by using the minimax value of each node:

MINIMAX(*s*)=

UTILITY(*s*)	If *s* is a terminal
$\max_{a \in Actions(s)}$ MINIMAX(RESULT(*s*,*a*))	If PLAYER(*s*)=MAX
$\min_{a \in Actions(s)}$ MINIMAX(RESULT(*s*,*a*))	If PLAYER(*s*)=MIN

9

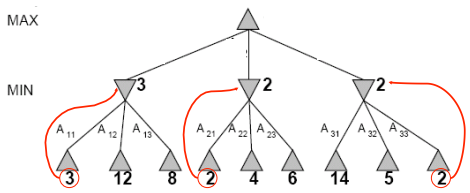
Two-Ply Game Tree



Definition: ply = turn of a two-player game

10

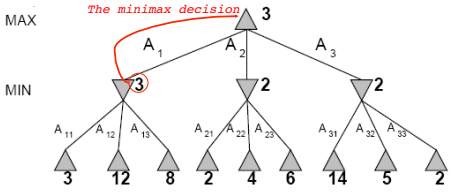
Two-Ply Game Tree



The minimax value at a min node is the minimum of backed-up values, because your opponent will do what's best for them (and worst for you).

11

Two-Ply Game Tree



Minimax maximizes the worst-case outcome for max.

12

Minimax Algorithm

```
function MINIMAX-DECISION(state) returns an action
  return arg max_{a ∈ Actions(s)} MIN-VALUE(RESULT(state,a))

function MAX-VALUE(state) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← ∞
  for each a in ACTIONS(state) do
    v ← MAX(v,MIN-VALUE(RESULT(state,a)))
  return v

function MIN-VALUE(state) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← ∞
  for a in ACTIONS(state) do
    v ← MIN(v,MAX-VALUE(RESULT(state,a)))
  return v
```

13

Properties of Minimax

- Minimax explores tree using DFS.
- Therefore:
  - Time complexity: $O(b^m)$ ☹
  - Space complexity: $O(bm)$ ☺

14

Problem of minimax search

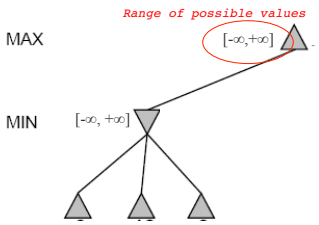
- Number of game states is exponential in the number of moves.
  - Solution: Do not examine every node
  - ==> Alpha-beta pruning
    - Remove branches that do not influence final decision
    - General idea: you can bracket the highest/lowest value at a node, even before all its successors have been evaluated

15

Alpha-Beta Pruning

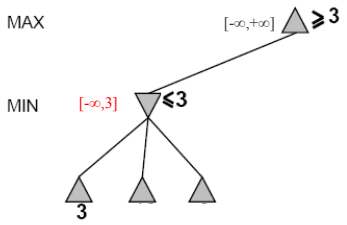
- α: the highest (i.e. best for Max) value possible
- β: the lowest (i.e. best for Min) value possible
- initially α and β are (-∞, ∞).

Alpha-Beta Example



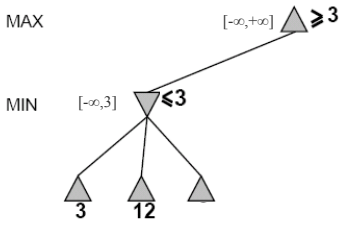
17

Alpha-Beta Example (continued)



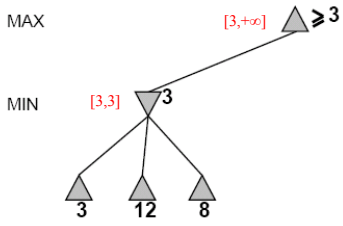
18

Alpha-Beta Example (continued)



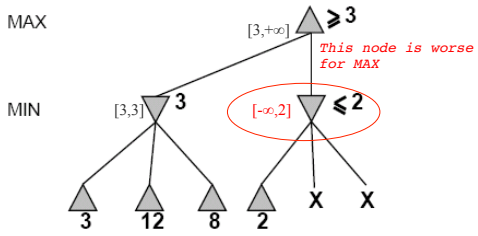
19

Alpha-Beta Example (continued)



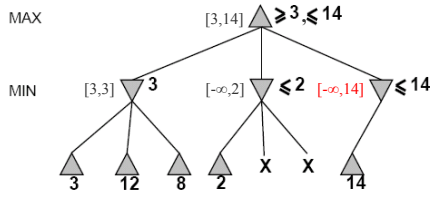
20

Alpha-Beta Example (continued)



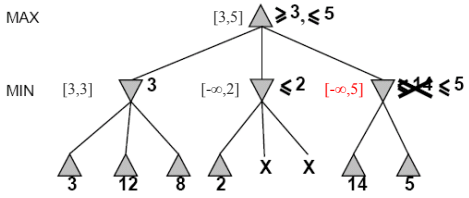
21

Alpha-Beta Example (continued)



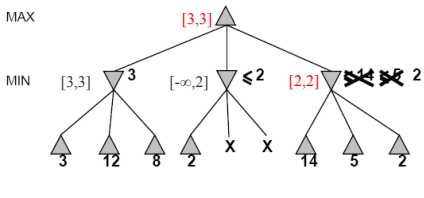
22

Alpha-Beta Example (continued)



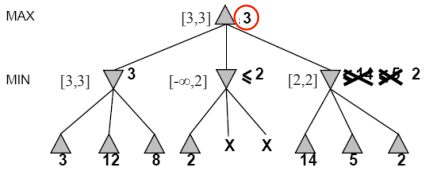
23

Alpha-Beta Example (continued)



24

Alpha-Beta Example (continued)



Alpha-Beta Algorithm

function ALPHA-BETA-SEARCH(*state*) **returns** *an action*
$v \leftarrow$ MAX-VALUE(*state*, $-\infty$, $+\infty$)
return the *action* in ACTIONS(*state*) with value *v*

---

function MAX-VALUE(*state*, α, β) **returns** *a utility value*
if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
$v \leftarrow -\infty$
for each *a* in ACTIONS(*state*) **do**
$v \leftarrow$ MAX(v, MIN-VALUE(RESULT(*state*,*a*), α, β))
if $v \geq \beta$ **then return** v
$\alpha \leftarrow$ MAX(α, v)
return v

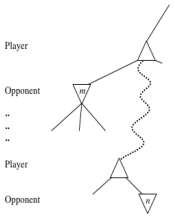
Alpha-Beta Algorithm

---

function MIN-VALUE(*state*, α, β) **returns** *a utility value*
if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
$v \leftarrow +\infty$
for each *a* in ACTIONS(*state*) **do**
$v \leftarrow$ MIN(v, MAX-VALUE(RESULT(*state*,*a*), α, β))
if $v \leq \alpha$ **then return** v
$\beta \leftarrow$ MIN(β, v)
return v

Alpha-beta pruning

- When enough is known about a node *n*, it can be pruned.



Final Comments about Alpha-Beta Pruning

- Pruning does not affect final results
- Entire subtrees can be pruned, not just leaves.
- Good move *ordering* improves effectiveness of pruning
- With "perfect ordering," time complexity is $O(b^{m/2})$
  - Effective branching factor of sqrt(b)
  - Consequence: alpha-beta pruning can look twice as deep as minimax in the same amount of time

Is this practical?

- Minimax and alpha-beta pruning still have exponential complexity.
- May be impractical within a reasonable amount of time.
- SHANNON (1950):
  - Terminate search at a lower depth
  - Apply heuristic evaluation function EVAL instead of the UTILITY function

Cutting off search

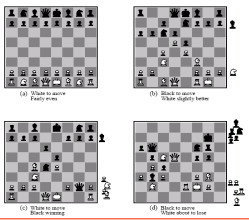
- Change:
  - **if** TERMINAL-TEST(*state*) **then return** UTILITY(*state*)

  into
  - **if** CUTOFF-TEST(*state*,*depth*) **then return** EVAL(*state*)
- Introduces a fixed-depth limit *depth*
  - Selected so that the amount of time will not exceed what the rules of the game allow.
- When cuttoff occurs, the evaluation is performed.

Heuristic EVAL

- Idea: produce an estimate of the expected utility of the game from a given position.
- Performance depends on quality of EVAL.
- Requirements:
  - EVAL should order terminal-nodes in the same way as UTILITY.
  - Fast to Compute.
  - For non-terminal states the EVAL should be strongly correlated with the actual chance of winning.

Heuristic EVAL example



Addition assumes independence

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

In chess:

w_1 *material* + w_2 *mobility* + w_3 *king safety* + w_4 *center control* + …

How good are computers…

- Let's look at the state of the art computer programs that play games such as chess, checkers, othello, go…

Checkers

- Chinook: the first program to win the world champion title in a competition against a human (1994)

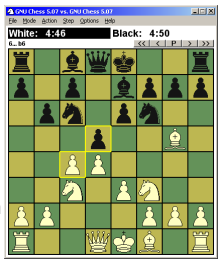
Chinook

- Components of Chinook:
  - Search (variant of alpha-beta). Search space has 10^{20} states.
  - Evaluation function
  - Endgame database (for all states with 4 vs. 4 pieces; roughly 40 billion positions).
  - Opening book - a database of opening moves
- Chinook can determine the final result of the game within the first 10 moves.
- Author has recently shown that several openings lead to a draw.

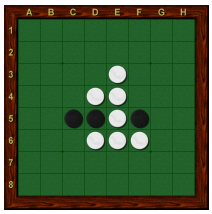
Jonathan Schaeffer, Neil Burch, Yngvi Bjornsson, Akihiro Kishimoto, Martin Muller, Rob Lake, Paul Lu and Steve Sutphen. "Checkers is Solved," Science, 2007. http://www.cs.ualberta.ca/~chinook/publications/solving_checkers.html

Chess

- 1997: Deep Blue wins a 6-game match against Garry Kasparov
- Searches using iterative deepening alpha-beta; evaluation function has over 8000 features; opening book of 4000 positions; end game database.
- FRITZ plays world champion, Vladimir Kramnik; wins 6-gam match.

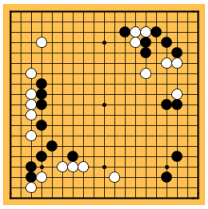


Othello



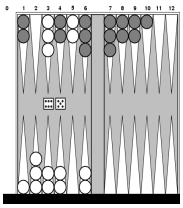
- The best Othello computer programs can easily defeat the best humans (e.g. Logistello, 1997).

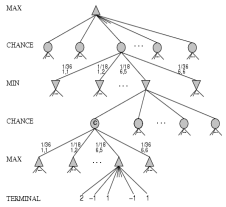
Go



- Go: humans still much better!

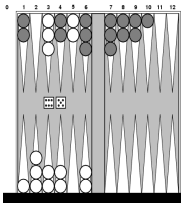
Games that include chance

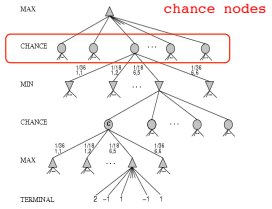




- Possible moves (5-10,5-11), (5-11,19-24),(5-10,10-16) and (5-11,11-16)

Games that include chance





- Possible moves (5-10,5-11), (5-11,19-24),(5-10,10-16) and (5-11,11-16)
- [1,1],…,[6,6] probability 1/36, all others - 1/18
- Can not calculate definite minimax value, only *expected* value

Expected minimax value

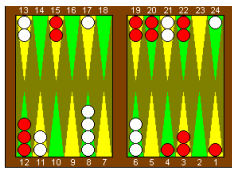
EXPECTIMINIMAX(*s*)=

UTILITY(*s*)	If *s* is a terminal
$\max_a$ EXPECTIMINIMAX(RESULT(*s*,*a*))	If *PLAYER(S)=MAX*
$\min_a$ EXPECTIMINIMAX(RESULT(*s*,*a*))	If *PLAYER(S)=MIN*
$\sum_r$ P(*r*) EXPECTIMINIMAX(RESULT(*s*,*r*))	If *PLAYER(S)=CHANCE*

r is a chance event (e.g., a roll of the dice).

These equations can be propagated recursively in a similar way to the MINIMAX algorithm.

TD-Gammon (Tesauro, 1994)



White's turn, with a roll of 4-4

World class program based on a combination of reinforcement Learning, neural networks and alpha-beta pruning to 3 plies.

Move analyses by TD-Gammon have lead to some changes in accepted strategies.

http://www.research.ibm.com/massive/tdl.html

43

Summary

- Games are fun
- They illustrate several important points about AI
  - Perfection is (usually) unattainable -> approximation
  - Uncertainty constrains the assignment of values to states

44