CS440

Assignment 1
Due Feb 11, 2019

Computer Science Department
Colorado State University

Feb 1, 2019

Preliminaries. Players A and B play a two-party game, taking turns, as explained in the following. Let $V = \{v_1, \ldots, v_6\}$ be the set of six vertices. The initial state of the game is empty graph as depicted in Figure 1(a). Suppose Player A picks blue and Player B picks red as their colors. Player A starts by selecting a pair of vertices (v_i, v_j) and draws a blue edge between them. Player B follows by selecting another pair of vertices (v_k, v_l) and draws a red edge between them. Player A takes the next turn and the game continues. In every turn, an already existing edge **cannot** be erased, redrawn, or overridden; just one new edge is *added* to the graph. Player A draws blue edges and Player B draws red edges.

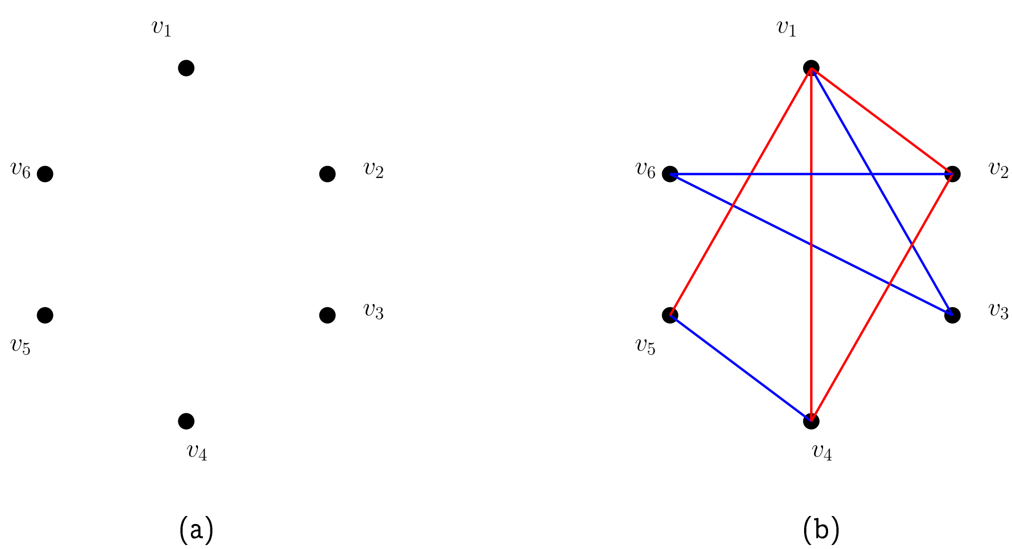

Figure 1: (a) Initial state of the game. No edges. (b) Player B wins because of the (red) unicolor triangle (v_1, v_2, v_4).

A triplet (v_i, v_j, v_k) is called a *unicolor triangle* if there exist edges (v_i, v_j), (v_i, v_k), and (v_j, v_k) of the same color. A unicolor triangle whose edges are blue is called a blue triangle, and similarly, a unicolor triangle whose edges are red is called a red triangle.

Game continues until either a blue triangle or a red triangle appears. If a blue triangle appears first, Player A wins. If a red triangle appears first, Player B wins the game. The goal of each player is to construct a unicolor triangle of their choice and obstruct creation of a unicolor triangle by the opponent. Figure 1(b) shows a sample red win state.

Existence of a winner [20 pts]. Prove that every game has an ultimate winner. To prove that, you need to show that in every game, no matter how it is played, there will eventually appear a unicolor triangle, which can be either blue or red. *Hint:* Consider $C = (V, E)$ the complete graph with 15 edges, each of which is arbitrarily colored blue or red. Prove that C contains at least one unicolor triangle.

Winning strategy [60 pts]. A player X is said to have a winning strategy if there exists a move for X in every turn, that no matter how the opponent plays, guarantees X will win. Write a program to decide the following mutually exclusive statements:

- Player A has a winning strategy.
- Player B has a winning strategy.
- Neither Player A nor Player B has a winning strategy.

Negated game [20 pts]. Suppose we negate the winning condition: if a blue triangle appears first, then Player B wins and if a red triangle appears first, then Player A wins. Repeat the winning strategy question above.

Bonus [30 pts]. Repeat all three questions for three players A, B, C with three colors blue, red, and green among seventeen vertices $V' = \{v_1, \ldots, v_{17}\}$.

Upload your answer on Canvas in one zip file or tarball. Include all the code/scripts you have written in your submission as well as (scanned) handwritten or typed answers.