CS 475: Performance Evaluation

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Analyzing Program Performance

- In empirical Computer Science, we plot functions describing the run time (or the memory use) of a program:
  - This can be as a **function of the input size**. We have seen this in e.g. cs320 or cs420, where we study polynomial and exponential (**monotonically growing**) sequential complexity.
  - In this class we also study program performance as a function of the number of processors.
    - In this case the functions are positive and, hopefully decreasing.
    - Also we plot speedup curves, which are usually asymptotic

Analyzing/Plotting Data

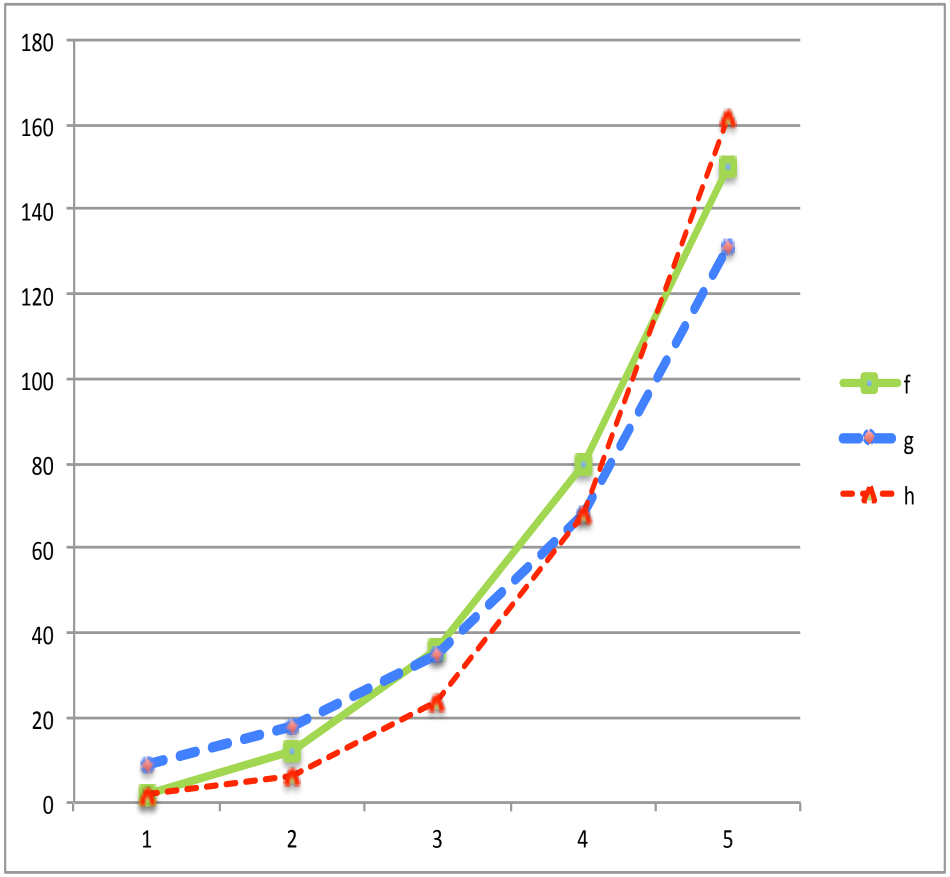
- When you run a program for a number of inputs (n) on a parallel machine with a number of processors (p), you end up with performance data sets. You want to characterize these in (a set of) functions:

  x: input size, y: performance or

  x: #processors, y: performance.
- To study (parallel) program's performance, we often use plotting tools
  - gnuplot, excel, matlab … (in these slides: excel)
- Let's look at increasing functions first.

Example: 3 data sets f, g and h

n	f(n)	g(n)	h(n)
1	2	9	2
2	12	18	6
3	36	35	24
4	80	68	68
5	150	131	162



What kinds of functions are f, g and h?

- exponential? which base?
- polynomial? which order?

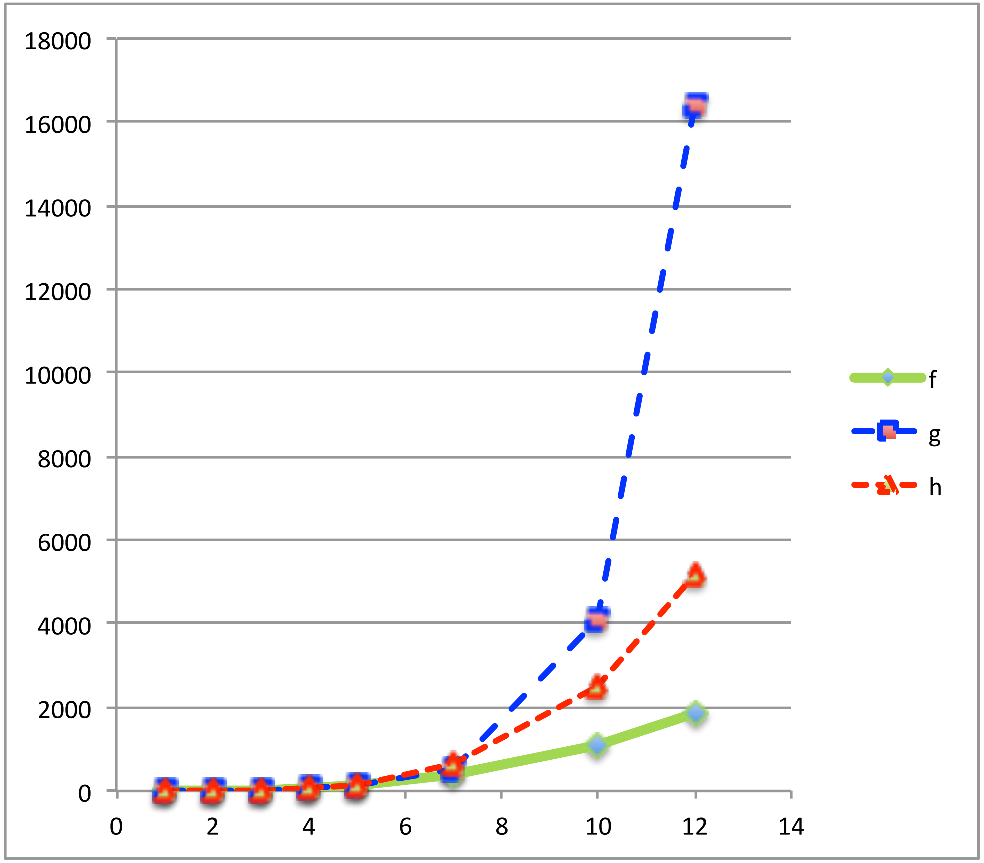
Hard / impossible to infer

Why are functions hard to infer?

- Two problems:
  - Very small domain (here 1..5)
    - Try to get a large data domain
  - Interpreting super-linear functions from plots is hard
    - All polynomials and exponentials **swoop up**

Larger domain

n	f(n)	g(n)	h(n)
1	2	9	2
2	12	18	6
3	36	35	24
4	80	68	68
5	150	131	162
7	400	520	624
10	1100	4106	2510
12	1872	16396	5196



Do you get a better idea now?
Which function may be polynomial, which exponential?
Still, not all clear (order, base...), h(n) may spike up later...

Straight Lines

We get the most information from **straight lines**!

- We can easily recognize a straight line ($y = ax+b$)
  - The **slope (a)** and **y intercept (b)** tells us all.

- **So we need to turn our data sets into straight lines.**
- This is easiest done using log-s, because they turn a **multiplicative factor into a shift** (y axis crossing b) , **and an exponential into a multiplicative factor** (slope a)

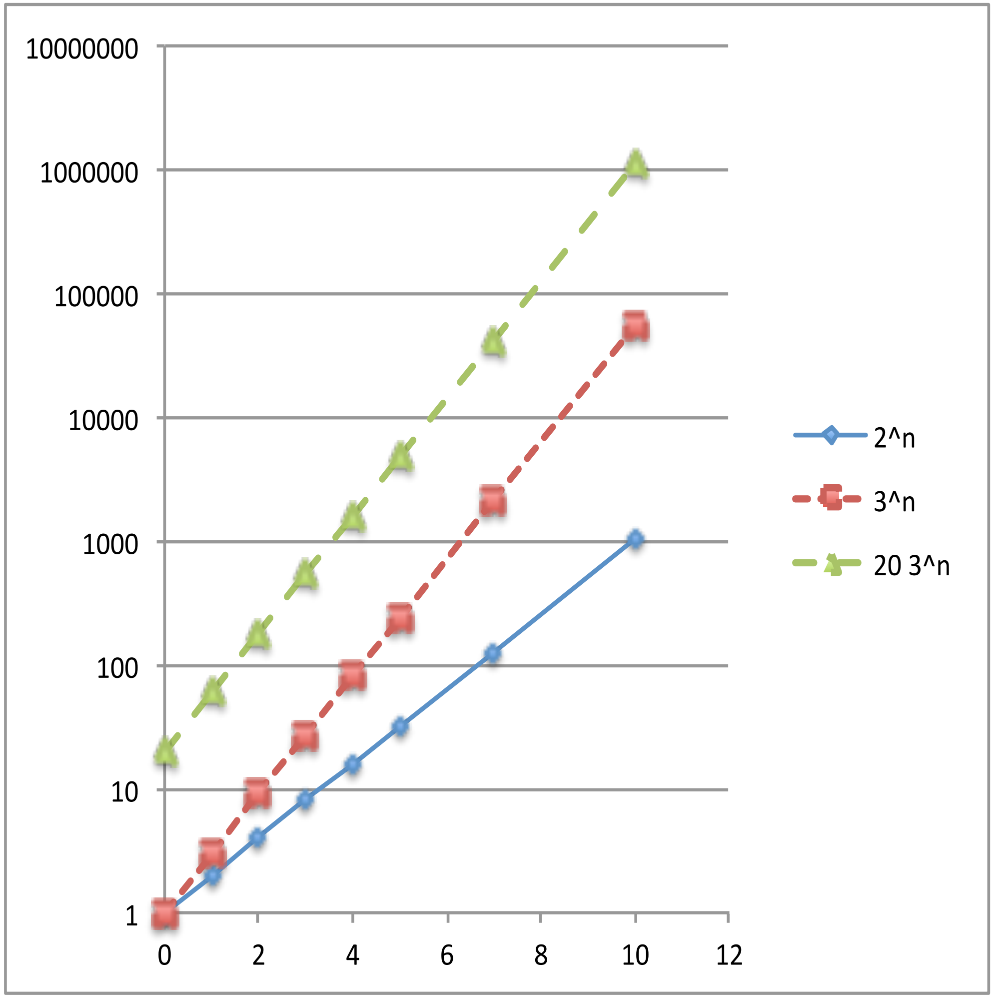
Exponential functions

- $\log(2^n) = n \log 2$ linear in n
- $\log(3^n) = n \log 3$ **angle** of the line: base of log

- $\log(4.3^n) = n \log 3 + \log 4$ *4 shifts **up**
- $\log((3^n)/4) = n \log 3 - \log 4$ /4 shifts **down**

Exponentials: semi-log plot

n	2^n	3^n	$20*3^n$
0	1	1	20
1	2	3	60
2	4	9	180
3	8	27	540
4	16	81	1620
5	32	243	4860
7	128	2087	41740
10	1024	56349	1126980



semi-log plot:
  y–axis on log scale
  x-axis linear
angle: base
shift: multiplicative factor

Polynomials

- What if we take the log of a polynomial?

  e.g. $f(n) = 5n^3$

  $$\log(f(n)) = \log(5n^3) = \log 5 + 3 \log(n)$$

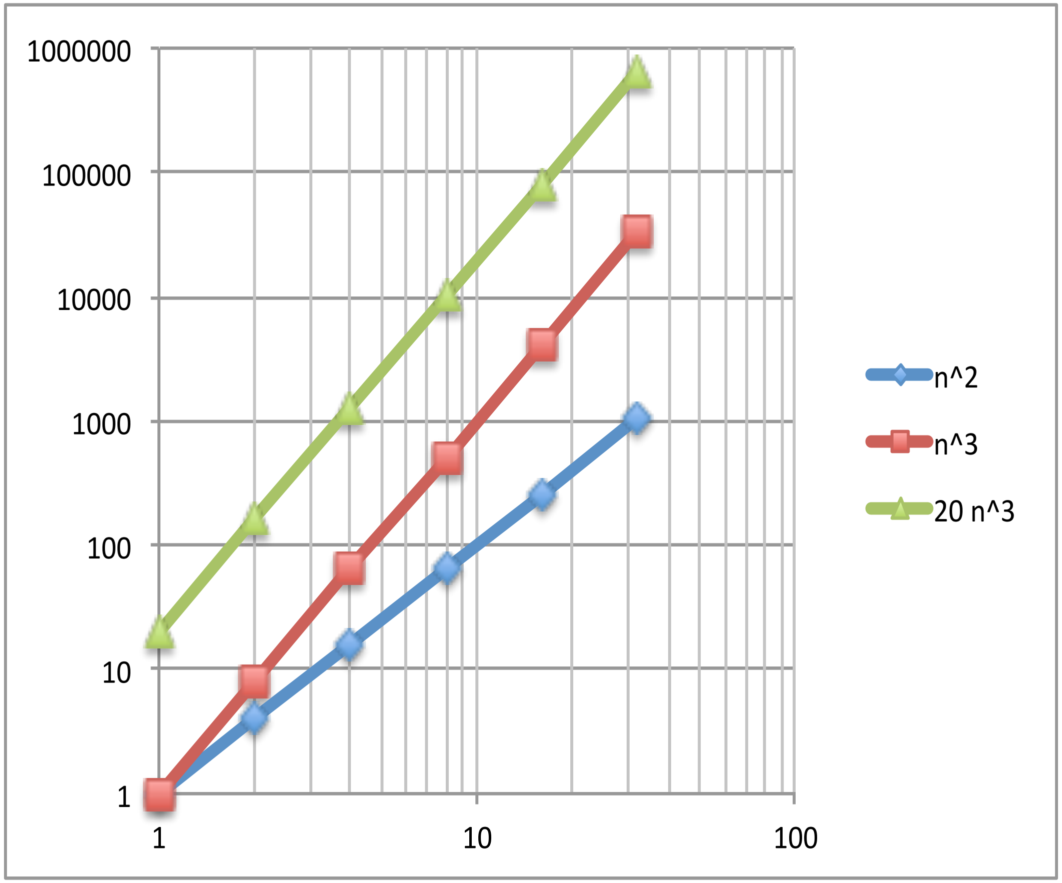
  not a straight line!

- But the **log of a polynomial is linear in log(n)**
- Therefore we need to plot polynomials on a log-log scale (both x and y axis logarithmic)

Polynomials: log-log plot

n	n^2	n^3	$20*n^3$
1	1	1	20
2	4	8	160
4	16	64	1280
8	64	512	10240
16	256	4096	81820
32	1024	32768	655360

angle: degree
shift: multiplicative factor



logs of sums

- Often we don't have a single factor in our function:
  - $3^n + 2^n$
  - $n^3 + n^2$
  - **Watch it:** log of sum is not sum of logs (what is?)
- Straight lines not completely straight anymore but **asymptotically straight:**

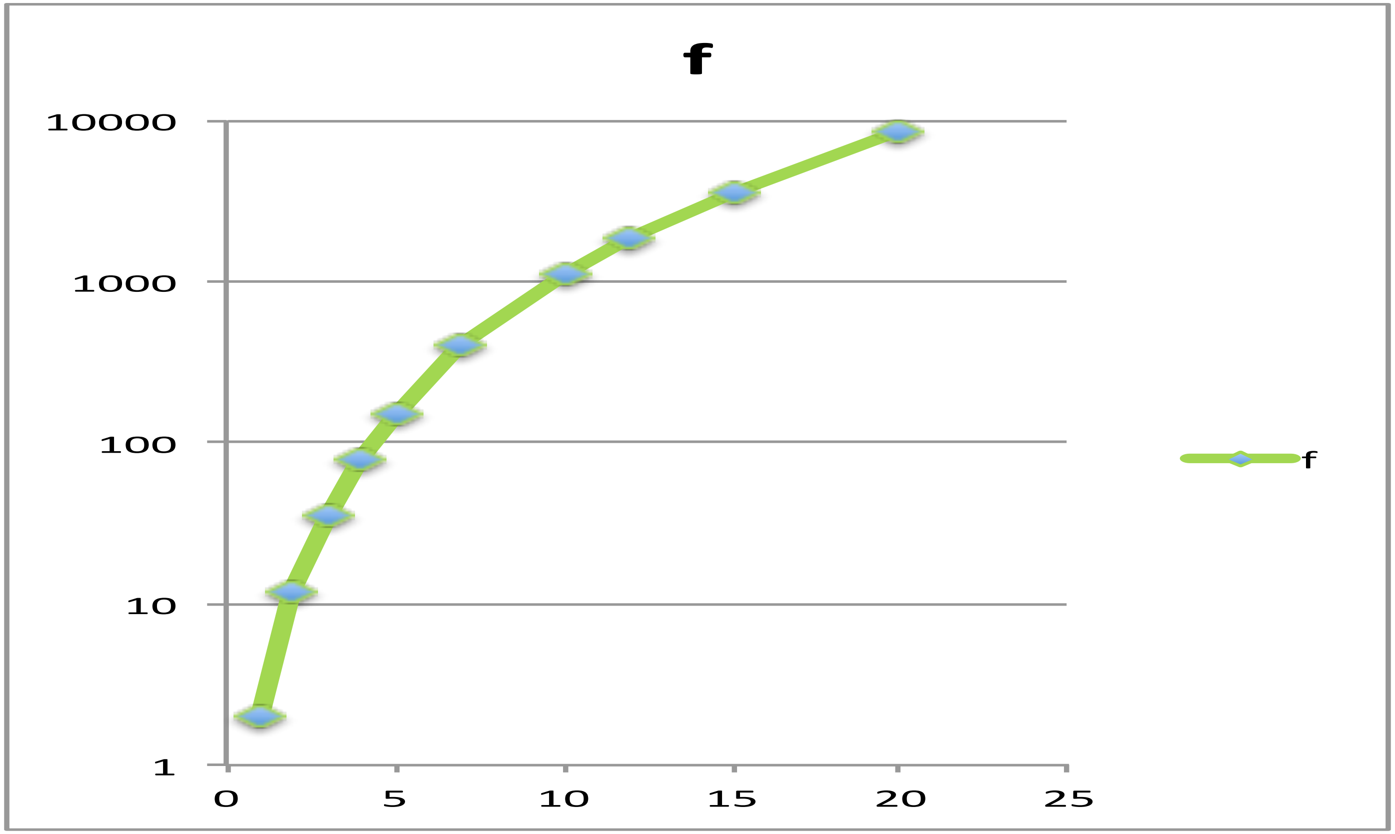
$$\log(3^n+2^n) = \log((1+(2/3)^n)3^n) = \log(1+(2/3)^n) + n\log(3)$$

$$\log(n^3+n^2) = \log((1+(1/n))n^3) = \log(1+(1/n)) + 3\log(n)$$

$\log(1+(2/3)^n)$ and $\log(1+(1/n))$ go to zero for large n

Back to the data: f

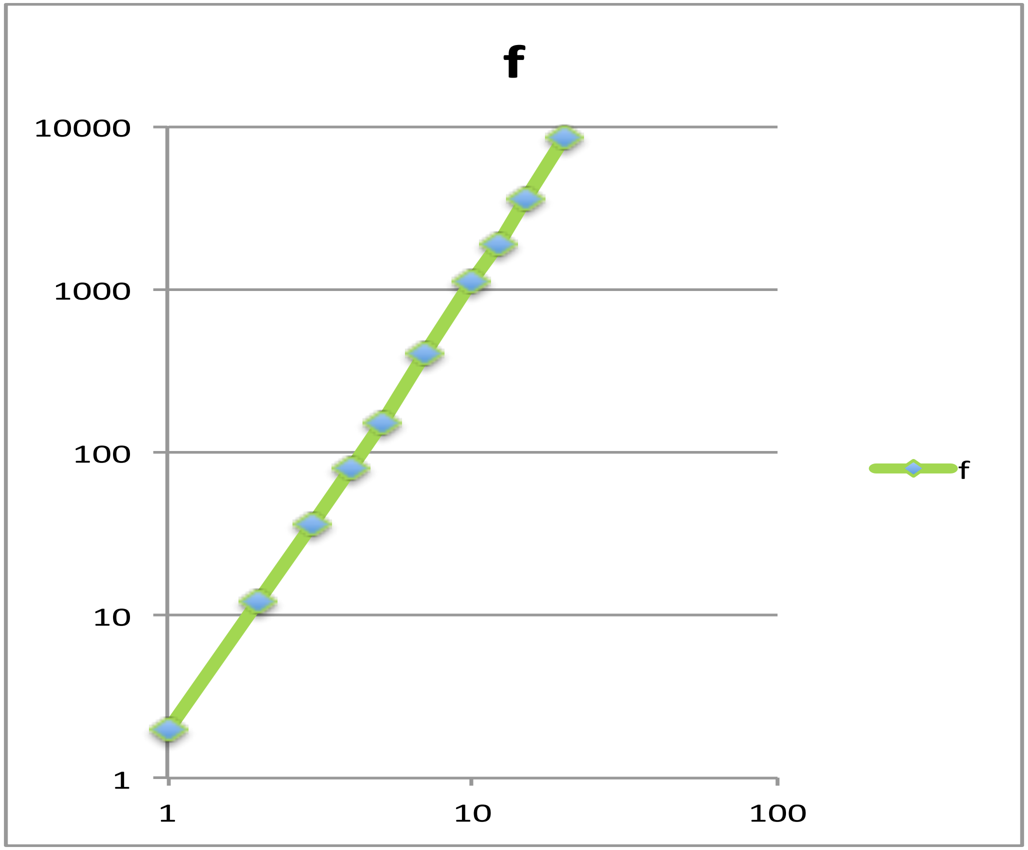
n	f(n)
1	2
2	12
3	36
4	80
5	150
7	400
10	1100
12	1872



The semi-log plot does not give a straight line, so f is not exponential

Is f polynomial?

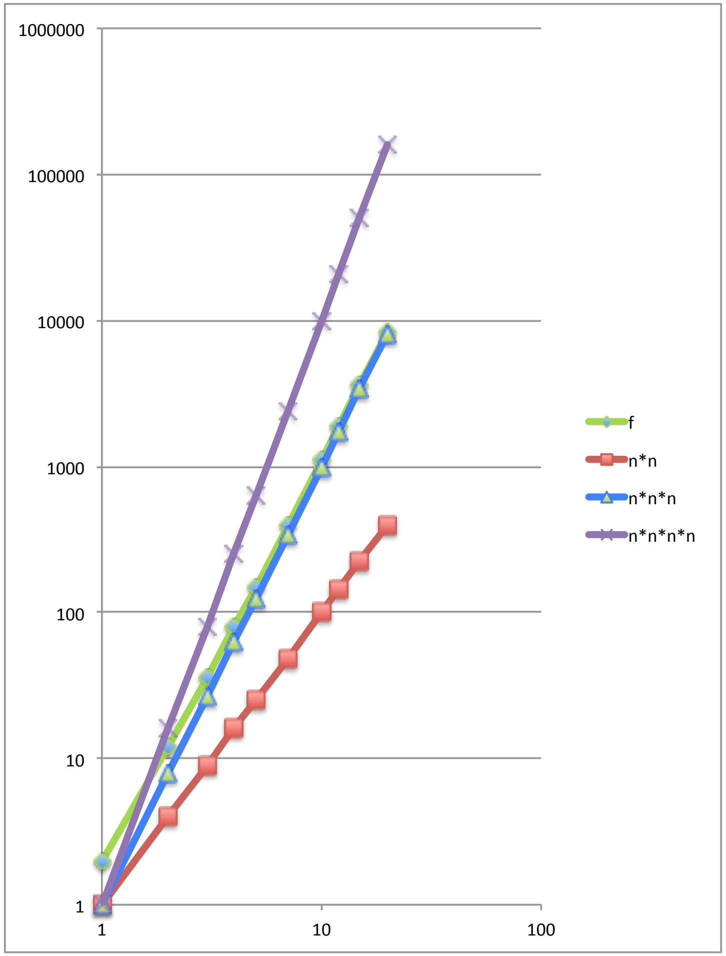
n	f(n)
1	2
2	12
3	36
4	80
5	150
7	400
10	1100
12	1872



YES! The log–log plot goes asymptotically to a straight line, so f is polynomial, but what is its leading term?

What is f's degree?

n	f(n)	n^2	n^3	n^4
1	2	1	1	1
2	12	4	8	16
3	36	9	27	81
4	80	16	64	256
5	150	25	125	625
7	400	49	343	2401
10	1100	100	1000	10000
12	1872	144	1728	20736

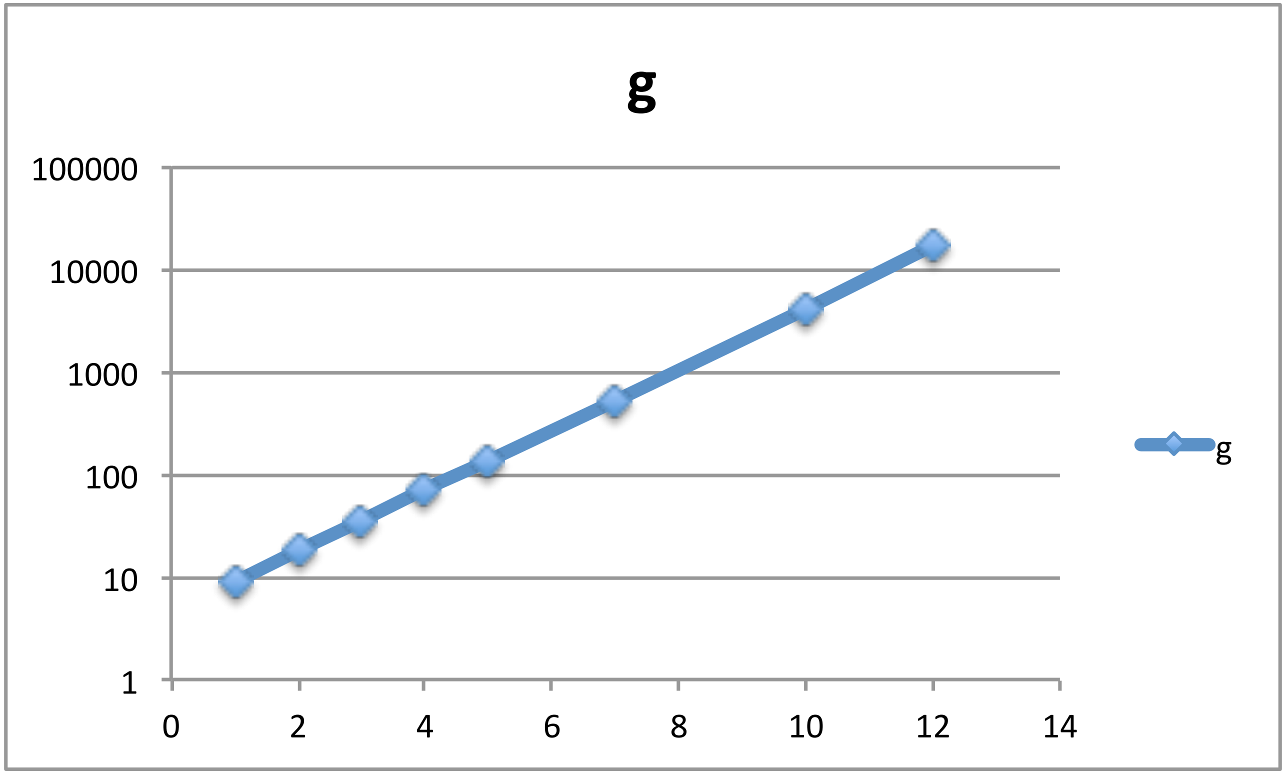


Compare with n, n^2,n^3,n^4

f is degree 3, no multiplicative factor (no shift up): $f(n)=n^3+$..
We usually only worry about the leading term.

How about g?

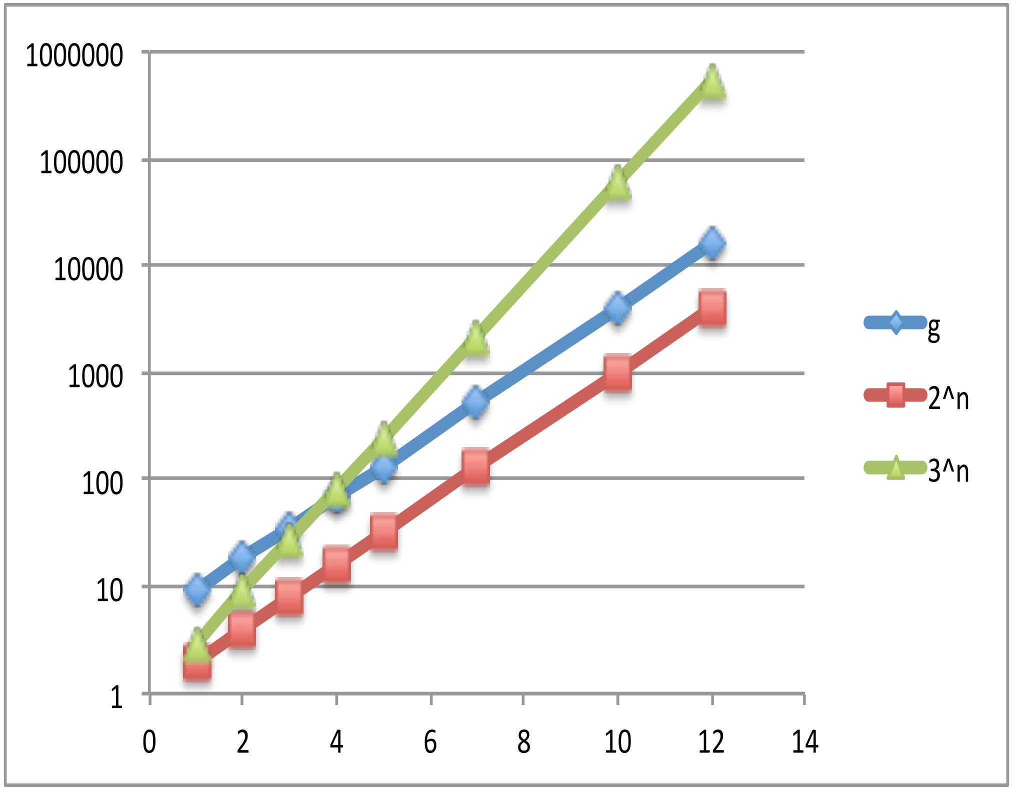
n	g(n)
1	9
2	18
3	35
4	68
5	131
7	520
10	4106
12	16396



g is linear on semi log plot so exponential
what base: compare to 2^n, 3^n

How about g?

n	g(n)	2^n	3^n
1	9	2	3
2	18	4	9
3	35	8	27
4	68	16	81
5	131	32	243
7	520	128	2187
10	4106	1024	59049
12	16396	4096	531441



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How about g?

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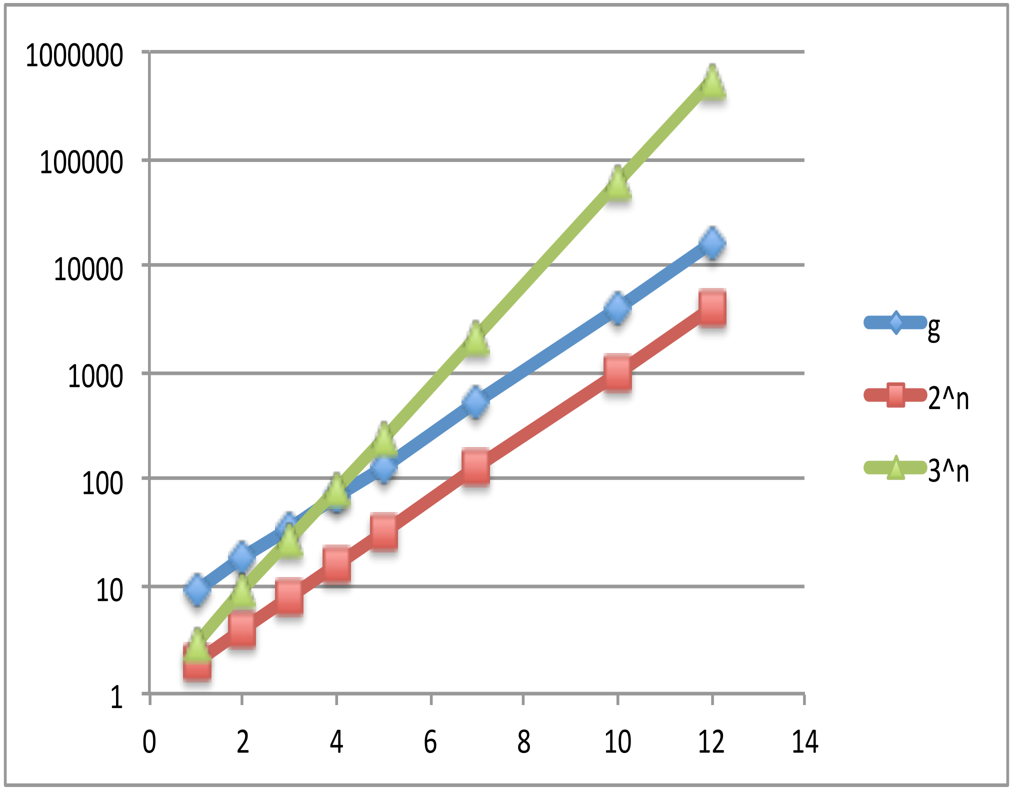

g: same slope as 2^n, but shifted up, factor 4

so $g(n) = 4.2^n + \ldots$

Decreasing functions

- This second class of functions can be used to represent running times of programs as a function of the number of processors.
- Ideally, these functions decrease **hyperbolically**
  - **f(p) = c/**p time to execute the program with p processors
  - **f(1) = c** sequential time
- But this is hardly ever the case. One of the reasons for this is that programs have inherently sequential parts, that do not speed up with more processors:
  - **f(p) = a + c/p** a: the sequential part, c: the parallelizable part

Plotting hyperbolic functions

- A simple way to turn T(p) = c/p into a straight line is to plot its reciprocal: **y = 1/T(p) = p/c**
  - This is a straight line with slope 1/c.
  - When analyzing parallel performance we scale this to y'=T(1)/T(p). If T(p) is the time it takes to execute a program with p processors, we call this the **speedup of the program**
- In the case of T(p) = a + c/p, the speedup is

$$S(p)=T(1)/T(p)= (a+c)/(a+c/p)$$

  - For a>0 this is not a straight, but a curve that grows and then flattens out to a constant (a+c)/a

Plotting Data: Summary

- Visually, a straight line conveys the most information.
  - If your data is not linear, massage it so that is linear, then deduce the original function.
- If y=f(x) is polynomial: log y is linear with log x

$$y = f(x) = a_0 + a_1 x + \cdots + a_n x^n \approx a_n x^n \quad \text{(asymptotically)}$$

$$\log y = \log a_n + n \log x$$

- If y=f(x) is exponential: log y is linear with x

$$y = f(x) = ba^x$$

$$\log y = \log b + x(\log a)$$

- In the case of T(p) = a + c/p, the speedup is

$$S(p)=T(1)/T(p)= (a+c)/(a+c/p)$$