CS 510 Final Exam
Spring 2010

NAME _____ **ANSWER KEY** ______

EID ________________________________

| Question | Max Points | Points |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 15 | |
| 6 | 15 | |
| 7 | 10 | |
| 8 | 10 | |
| 9 | 10 | |
| TOTAL | 100 | |

Question 1: Closest Point (10 Points)

A ray in 3-D space is defined by a point of origination $P$ and a direction $V$.

$$P = \begin{vmatrix} a \\ b \\ c \end{vmatrix} \qquad V = \begin{vmatrix} u \\ v \\ w \end{vmatrix} \qquad V \cdot V = 1$$

Write out the precise algebraic expression for the point $T$ lying on this ray closest to a point $Q$

$$Q = \begin{vmatrix} x \\ y \\ z \end{vmatrix}$$

Please briefly explain the process you use to arrive at your answer.

First, for the sake of simplicity, shift to origin to the point P, thus we have a new version of the point Q in this transformed coordinate system.

$$Q' = \begin{vmatrix} x-a \\ y-b \\ z-c \end{vmatrix}$$

The distance (signed) from origin to $Q'$ in the direction defined by $V$ may be expressed as the projection of $Q'$ onto the ray defined by the unit vector $V$

$$d = Q' \cdot V = (x-a)u + (y-b)v + (z-c)w$$

So, putting it all together, the point $T$ is arrived at by moving a distance $d$ from the point $P$ in the direction $V$.

$$T = \big((x-a)u + (y-b)v + (z-c)w\big)V + P$$

Question 2: Normal to a Plane (10 Points)

Consider the triangle defined by the following 3 points

$$P_1 = \begin{vmatrix} 2 \\ 2 \\ 3 \end{vmatrix} \quad P_2 = \begin{vmatrix} 3 \\ 2 \\ 2 \end{vmatrix} \quad P_3 = \begin{vmatrix} 2 \\ 2 \\ 2 \end{vmatrix}$$

What is the normal vector associated with this triangle?

To start, find two vectors that both lie in the plane of the triangle. This could be done in a variety of ways, but for these particular values it will be easiest to work this example by hand if we choose the third point to serve as the origin for the vectors.

$$V = P_1 - P_3 \qquad U = P_2 - P_3$$

$$V = \begin{vmatrix} 2 \\ 2 \\ 3 \end{vmatrix} - \begin{vmatrix} 2 \\ 2 \\ 2 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} \qquad U = \begin{vmatrix} 3 \\ 2 \\ 2 \end{vmatrix} - \begin{vmatrix} 2 \\ 2 \\ 2 \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix}$$

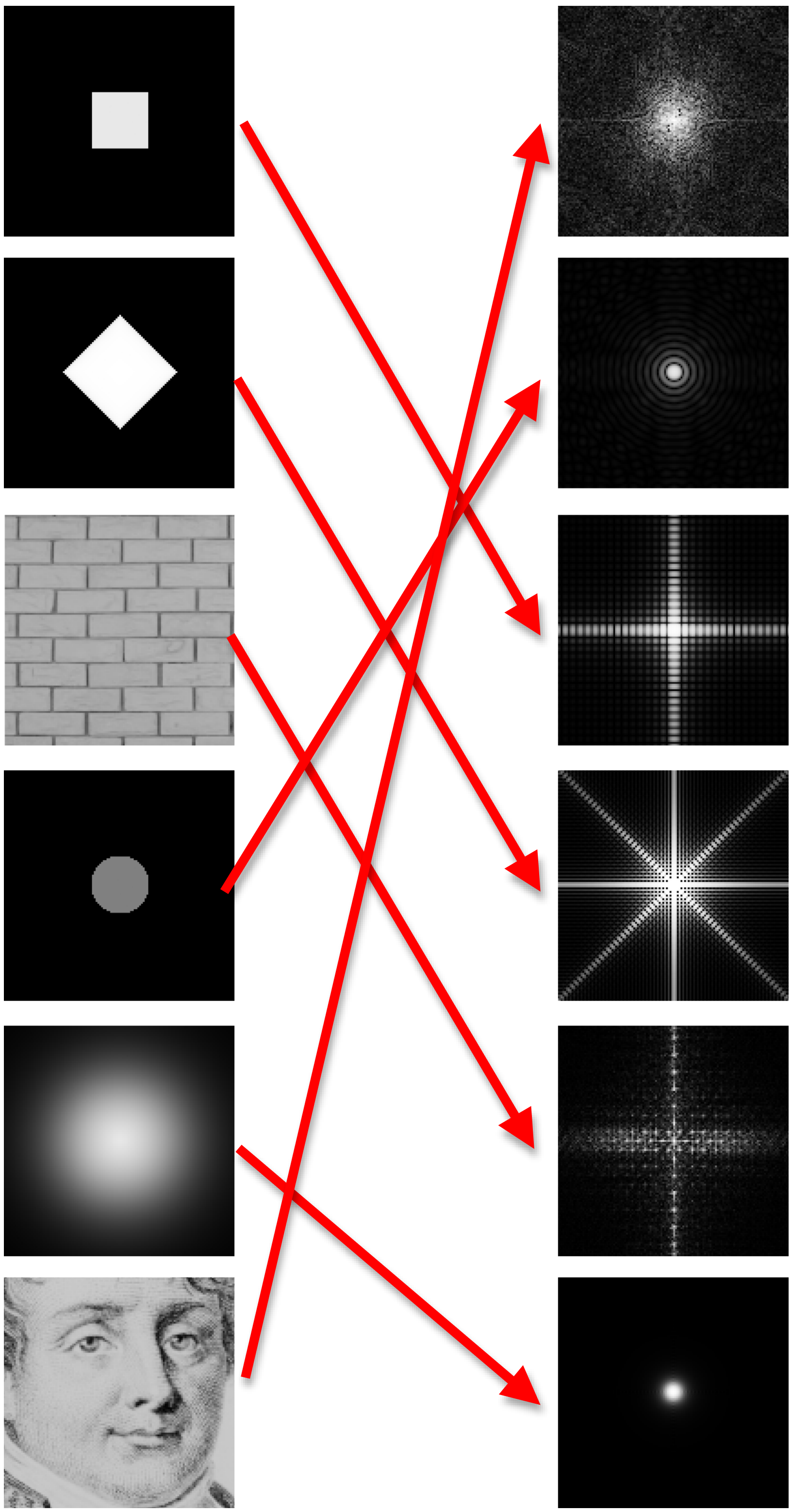
From this point, you can either solve the problem by inspection or algebraically. By inspection, note the sides of an equivalent triangle has as two of its sides unit length vectors along the x and z axes. Therefore, the plane is the x-z plane and the unit normal points in the y direction.

Algebraically

$$N = V \times U = \begin{vmatrix} 0 \\ 0 \\ 1 \end{vmatrix} \times \begin{vmatrix} 1 \\ 0 \\ 0 \end{vmatrix} = \begin{vmatrix} 0*0-0*1 \\ 0*0-1*1 \\ 0*0-1*0 \end{vmatrix} = \begin{vmatrix} 0 \\ -1 \\ 0 \end{vmatrix}$$

Note for this problem, whether to use $V$ cross $U$ or vice versa is not specified, so the sign on the $y$ component may be either plus or minus.

Question 3: There is an excellent Java Applet that shows images and their associated Fourier transforms (http://www.brainflux.org/java/classes/FFT2DApplet.html). Here are six images and their associated Fourier transforms. Draw lines to match up the corresponding images and their transforms. Note only the magnitude of the Fourier transform is shown. (10 Points)



Question 4: Consider convolving an image twice with the following mask (10 Points)

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

In the space provided below, write in the values for a mask that will produce an equivalent result when convolved only once with an image.

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 2 | 3 | 2 | 1 | 0 |
| 0 | 2 | 4 | 6 | 4 | 2 | 0 |
| 0 | 3 | 6 | 9 | 6 | 3 | 0 |
| 0 | 2 | 4 | 6 | 4 | 2 | 0 |
| 0 | 1 | 2 | 3 | 2 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Question 5: Comparing Vectors (15 Points)

Consider two vectors:

$$V = \begin{vmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \end{vmatrix} \qquad U = \begin{vmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_n \end{vmatrix}$$

Part 1: Write down precisely the equation for the L2 distance between $V$ and $U$. (3 Points)

$$d_{L2}(V,U) = \sqrt{\sum_{i=1}^{n} (v_i - u_i)^2}$$

Part 2: Write down precisely the equation for the correlation between $V$ and $U$. (4 Points)

$$c(V,U) = \frac{\sum_{i=1}^{n} (v_i - \bar{v})(u_i - \bar{u})}{\sqrt{\sum_{i=1}^{n} (v_i - \bar{v})^2}\sqrt{\sum_{i=1}^{n} (u_i - \bar{u})^2}} \quad \text{where} \quad \bar{v} = \frac{1}{n}\sum_{i=1}^{n} v_i \quad \text{and} \quad \bar{u} = \frac{1}{n}\sum_{i=1}^{n} u_i$$

Part 3: Prove that under a special case, minimizing L2 distance is equivalent to maximizing correlation. (8 Points)

$$Let \quad \bar{v} = \bar{u} = 0 \quad and \quad |v| = \sum_{i=1}^{n} v_i^2 = 1 \quad and \quad |u| = \sum_{i=1}^{n} u_i^2 = 1$$

$$\begin{aligned} d_{L2}(V,U) &= \sqrt{\sum_{i=1}^{n} (v_i - u_i)^2} \\ &= \sqrt{\sum_{i=1}^{n} v_i^2 + u_i^2 - 2v_i u_i} \\ &= \sqrt{\sum_{i=1}^{n} v_i^2 + \sum_{i=1}^{n} u_i^2 - 2\sum_{i=1}^{n} v_i u_i} \\ &= \sqrt{2 - 2\sum_{i=1}^{n} v_i u_i} \\ &= \sqrt{2 - 2c(V,U)} \end{aligned}$$

Question 6: Principal Component Analysis (15 Points)

To illustrate the basic concepts, consider the following rather simple data matrix consisting of 2 3-D points.

$$X = \begin{vmatrix} 5 & 15 \\ 10 & 30 \\ 20 & 60 \end{vmatrix}$$

Part 1: Write down the 'centered' version of X (5 Points)

$$X = \begin{vmatrix} -5 & 5 \\ -10 & 10 \\ -20 & 20 \end{vmatrix}$$

Part 2: Write down the scatter matrix associated with the Eigen System that one would solve to find the principal components. Recall the scatter and covariance matrices differ only by a scale factor in this context and the distinction does not matter for this question. (5 Points)

$$C = XX^T = \begin{vmatrix} -5 & 5 \\ -10 & 10 \\ -20 & 20 \end{vmatrix} \begin{vmatrix} -5 & -10 & -20 \\ 5 & 10 & 20 \end{vmatrix} = \begin{vmatrix} 50 & 100 & 200 \\ 100 & 200 & 400 \\ 200 & 400 & 800 \end{vmatrix}$$

Part 3: This is a particularly simple example where it is possible to state the direction of this first principal component by inspection. What is the first principal component's direction? (5 Points)

$$V_1 = \begin{vmatrix} 5 \\ 10 \\ 20 \end{vmatrix} = \begin{vmatrix} 1 \\ 2 \\ 4 \end{vmatrix}$$

Question 7: Hough Transforms (10 Points)

The Hough space used to find lines in an image is parameterized by two values

$\rho$
$\theta$

Part 1: First state in plain English what each of represents. Draw a picture if you think it will help you with your explanation. (5 Points)

The first parameter is the distance from the line (infinite) to the origin.

The second parameter is the orientation of the line measured, for example measured in radians with zero being horizontal.

Part 2: Consider an edge passing through pixel $P_e$ with a gradient direction given by a vector $V$:

$$P_e = \begin{vmatrix} 15 \\ 10 \end{vmatrix} \quad V = \begin{vmatrix} 3 \\ 4 \end{vmatrix}$$

Where would this edge vote in Hough space? You need not give angles in radians or in degrees, it is sufficient to describe them in terms of trigonometric functions such as arc tangent. (5 Points)

There is some flexibility with respect to the angle as to whether it is the orientation of a vector normal to the line, i.e. the gradient direction, or the line itself. It does not matter much so lone as one is consistent. Here, let us define the orientation as being for the normal to the line:

$$\theta = \tan^{-1}\left( {}^{4}\!/_{3} \right)$$

The other parameter is the distance from the origin of the infinite line passing through the point where the orientation of the line is perpendicular to the gradient. Thus, this is actually a question about dot products. Take the dot product between the unit length normal vector for the line and the point itself to find the second Hough parameter:

$$\hat{N} = \begin{vmatrix} {}^{3}\!/_{5} \\ {}^{4}\!/_{5} \end{vmatrix} \quad \rho = \begin{vmatrix} {}^{3}\!/_{5} \\ {}^{4}\!/_{5} \end{vmatrix} \cdot \begin{vmatrix} 15 \\ 10 \end{vmatrix} = \frac{45 + 40}{5} = 17$$

Question 8: Region Segmentation. (10 Points)

Here is a 20 by 20 pixel grey-scale image. Consider the region containing the upper left pixel as defined by a flood fill algorithm with a threshold (inclusive) of 10 grey levels. Show this region by shading all pixels in this region. Be as precise as you can – every pixel counts.

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 |
| 255 | 254 | 251 | 238 | 145 | 129 | 211 | 248 | 249 | 130 | 131 | 131 | 131 | 131 | 132 | 130 | 127 | 211 | 245 | 255 |
| 255 | 251 | 255 | 161 | 117 | 106 | 113 | 255 | 255 | 239 | 115 | 115 | 115 | 114 | 115 | 115 | 110 | 130 | 215 | 255 |
| 255 | 248 | 255 | 143 | 117 | 107 | 93 | 214 | 255 | 255 | 122 | 115 | 114 | 116 | 114 | 115 | 109 | 109 | 188 | 255 |
| 255 | 248 | 255 | 135 | 109 | 101 | 90 | 153 | 255 | 255 | 146 | 115 | 116 | 115 | 114 | 113 | 105 | 104 | 179 | 255 |
| 255 | 248 | 255 | 173 | 96 | 92 | 84 | 152 | 255 | 255 | 111 | 114 | 116 | 115 | 113 | 107 | 102 | 102 | 179 | 255 |
| 255 | 249 | 255 | 242 | 91 | 83 | 88 | 226 | 255 | 222 | 100 | 123 | 142 | 136 | 104 | 80 | 83 | 101 | 179 | 255 |
| 255 | 167 | 241 | 255 | 227 | 162 | 227 | 255 | 227 | 143 | 154 | 151 | 143 | 136 | 131 | 124 | 118 | 102 | 179 | 255 |
| 255 | 105 | 90 | 149 | 213 | 255 | 255 | 234 | 134 | 145 | 150 | 148 | 142 | 134 | 129 | 123 | 117 | 118 | 178 | 255 |
| 255 | 105 | 74 | 73 | 104 | 255 | 255 | 245 | 133 | 141 | 142 | 140 | 139 | 133 | 126 | 121 | 117 | 116 | 179 | 255 |
| 255 | 100 | 68 | 93 | 100 | 240 | 255 | 255 | 246 | 146 | 133 | 135 | 132 | 129 | 123 | 119 | 114 | 116 | 178 | 255 |
| 255 | 193 | 241 | 255 | 255 | 255 | 255 | 255 | 255 | 245 | 148 | 127 | 128 | 125 | 120 | 116 | 110 | 116 | 179 | 255 |
| 255 | 249 | 250 | 212 | 197 | 190 | 181 | 208 | 255 | 255 | 234 | 117 | 122 | 118 | 117 | 111 | 109 | 116 | 179 | 255 |
| 255 | 249 | 183 | 183 | 192 | 192 | 189 | 174 | 200 | 255 | 255 | 136 | 114 | 114 | 111 | 109 | 108 | 115 | 179 | 255 |
| 255 | 248 | 164 | 180 | 184 | 184 | 178 | 173 | 182 | 255 | 255 | 123 | 111 | 110 | 110 | 109 | 107 | 113 | 178 | 255 |
| 255 | 248 | 175 | 173 | 175 | 176 | 175 | 171 | 193 | 255 | 230 | 102 | 109 | 109 | 108 | 105 | 103 | 112 | 178 | 255 |
| 255 | 248 | 234 | 189 | 165 | 166 | 171 | 200 | 247 | 255 | 133 | 103 | 105 | 105 | 103 | 100 | 98 | 111 | 180 | 255 |
| 255 | 235 | 214 | 193 | 188 | 188 | 188 | 188 | 188 | 154 | 111 | 113 | 112 | 112 | 111 | 110 | 110 | 146 | 195 | 255 |
| 255 | 245 | 213 | 187 | 178 | 179 | 178 | 179 | 179 | 178 | 179 | 179 | 179 | 178 | 179 | 179 | 180 | 195 | 228 | 255 |
| 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 |

Note that is solution assumes a 4-connected neighborhood rather than 8-connected. Consistent 8-connected solutions are also fine.

Question 9: Object Recognition. (10 Points)

Part 1: In document retrieval the phrase 'bag of words' arises. What does the phrase mean in this context? Please backup your answer by applying the essential idea to the following tiny fragment of a document. (5 Points)

*This land is your land, this land is my land*
*From California to the New York Island*
*From the Redwood Forest to the Gulf Stream waters*
*This land was made for you and me.*

The essential idea is to place all the words in a document, perhaps minus what are sometimes called stop words such as 'a' and 'the' into an unordered set, a bag, and then count how often words appear. These counts in turn become a feature vector that characterizes the document and which can be used for document retrieval. Here are the word counts for the example document fragment. Note exactly how you decide which words are 'stop' words is a matter of taste, within reason, for this example.

| Word | Count | Word | Count | Word | Count | Word | Count |
|---|---|---|---|---|---|---|---|
| california | 1 | island | 1 | made | 1 | you | 1 |
| forest | 1 | land | 5 | redwood | 1 | your | 1 |
| from | 2 | made | 1 | stream | 1 | | |
| gulf | 1 | island | 1 | this | 3 | | |
| is | 2 | land | 5 | waters | 1 | | |

Part 2: Why would 'bag of words' be a concept of interest to researchers interested in identifying objects in images? (5 Points)

Recall that in the lecture 'Recent Advances in Object Recognition', Stephen O'Hara walked us through a modern approach to object recognition where first a number of focus-of-attention features were found in an image, and then these were treated more-or-less like words in so much as features were grouped into categories and then how often a feature of a given type appeared in an image was counted. These feature counts in turn can become the basis for an object recognition algorithm.