Getting Beyond Distance: (Stats) Cross-Correlation & (EE) Correlation

CS 510
Lecture #9
February 18, 2015

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Pearson's Correlation

Recall from the previous lecture …

$$\frac{\sum_{x,y}\left(A(x,y)-\bar{A}\right)\left(B(x,y)-\bar{B}\right)}{\sqrt{\sum_{x,y}\left(A(x,y)-\bar{A}\right)^2}\sqrt{\sum_{x,y}\left(B(x,y)-\bar{B}\right)^2}}$$

This is a very important equation…

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Assumptions of Correlation

$$\frac{\sum_{x,y}\left(A(x,y)-\bar{A}\right)\left(B(x,y)-\bar{B}\right)}{\sqrt{\sum_{x,y}\left(A(x,y)-\bar{A}\right)^2}\sqrt{\sum_{x,y}\left(B(x,y)-\bar{B}\right)^2}}$$

- Two signals vary linearly
  - Constant shift to either signal has no effect.
  - Increased amplitude has no effect.
- This minimizes sensitivity to:
  - changes in (overall) illumination
  - offset or gain.

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Computing Correlation

- A constant added to A does not change its correlation to any other signal, so
  - Let's subtract average A from A()
  - Let's subtract average B from B()
  - The mean of both signals is now zero
  - Then correlation reduces to:

$$\frac{A \cdot B}{\sqrt{\sum_{x,y}\left(A(x,y)-\bar{A}\right)^2}\sqrt{\sum_{x,y}\left(B(x,y)-\bar{B}\right)^2}}$$

Know and love the dot product.

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Computing Correlation (II)

- For zero-mean signals, we can scale them without changing their correlation scores
  - Multiply A by the inverse of its length
  - Multiply B by the inverse of its length
  - Both signals are now unit length
  - Then correlation reduces to:

$$A \cdot B$$

Know and love the dot product. ☺

- Gives rise to 'Correlation Space'.

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Correlation Space

- Why zero-mean & unit-length your images?
- Consider, for example, database retrieval
  - Compare new image A …
  - with many images in database.
  - When database images are stored in their zero-mean & unit-length form, then
  - Preprocess A (zero-mean, unit-length)
  - Compute dot products

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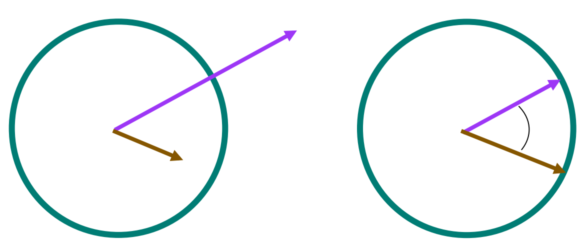
Correlation Space (II)

- New idea: image as a point in an N dimensional space
    - N = width x height
- Zero-mean & unit-length images lie on an N-1 dimensional "correlation space" where the dot product equals correlation.
    - This is a highly non-linear projection.
    - Points lie on an N-1 surface within the original N dimensional space.
- So consider points in 3-D ….

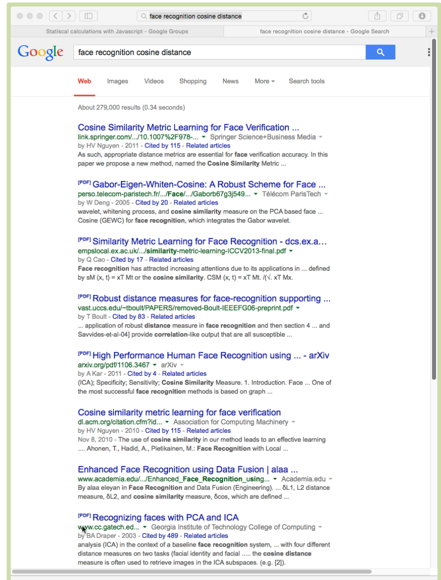
Correlation Space (II)

- Subtracting mean - translation.
- Length one - project onto sphere.
- Correlation is then:
    - Cosine of angle between vectors (points).



Useful?

- Yes
- Very commonly used.
- For example
  - Face Rec.
- Google
  - 279,000 hits



Useful Connection …

- Euclidean distance is inversely proportional to correlation in correlation space.

$$\sqrt{\sum_{x,y}\left(A[x,y]-B[x,y]\right)^2} = \sqrt{\sum_{x,y}A[x,y]^2+\sum_{x,y}B[x,y]^2-2A[x,y]B[x,y]}$$

$$= \sqrt{1+1-2\sum_{x,y}A[x,y]B[x,y]}$$

$$= \sqrt{2-2A\cdot B}$$

$$= \sqrt{2-2Corr(A,B)}$$

- Nearest-neighbor classifiers in correlation space maximize correlation / minimized L_2 norm

Limitations

- To match images this way, they must be
  - The same width & height
  - In correspondence : coordinates match
- More importantly, objects in the scene must
  - Be in the same location
  - Be at the same scale
  - Be at the same orientation
  - Be seen from the same viewpoint

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New Goal: Find a small image within a larger one

- The image above is a small piece of the image to the right. But from where?

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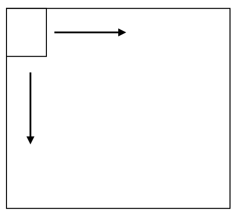
Brute-Force Translation Invariance

To find a small image in a large one, "slide" the small one across the large, computing Pearson's correlation at every possible position.



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Statistical Cross-Correlation

- The process of "slide & correlate" is called cross-correlation
- Complexity is O(nm)
  - N = # of pixels in image (w×h)
  - M = # of pixels in the template (w×h)
- Highly parallel (every position can be computed independently)
- Still sensitive to
  - Rotation
    - in-plane
    - out-of-plane
  - Scale
  - Perspective

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Computing Cross-Correlation

- In cross-correlation, the mask is correlated repeatedly to image windows
  - zero-mean & unit length the mask
  - zero-mean & unit length the image
  - compute the sliding dot product

This is *almost* convolving the image with the mask

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Naming conventions

- In Engineering, convolving a normalized mask with the source image is called correlation
  - Is this exactly the same as Pearson's correlation?
  - Why or why not?
- This is the most common definition of correlation in image processing texts

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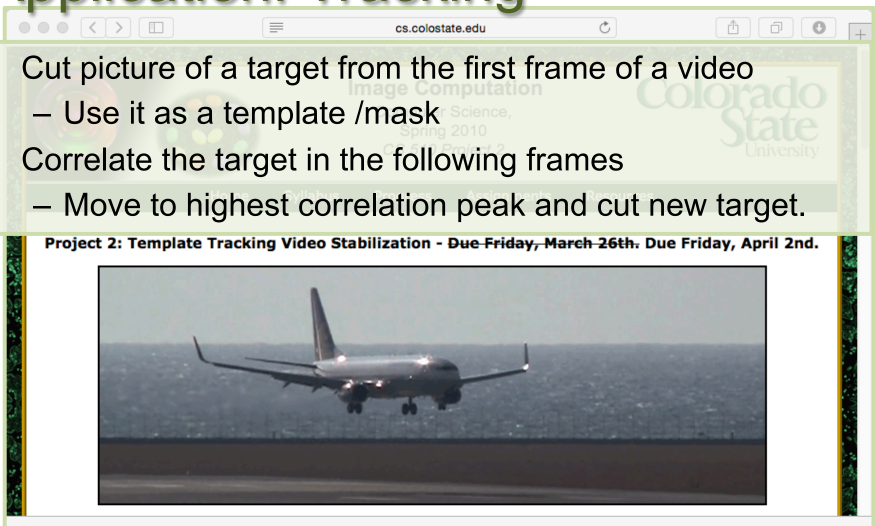
Application: Tracking

- Cut picture of a target from the first frame of a video
  - Use it as a template /mask
- Correlate the target in the following frames
  - Move to highest correlation peak and cut new target.



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Add Scale Correction

ECE 695 Project: OBJECT TRACKING IN VIDEOS, Arun Anbumani, December 18, 2013

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Application: Mosaicing

- Take several, overlapping images from a translating camera
  - Camera cannot move along optical axis
- Correlate the whole images to each other
  - Find location where they match the best
  - Stitch them into a single, larger image

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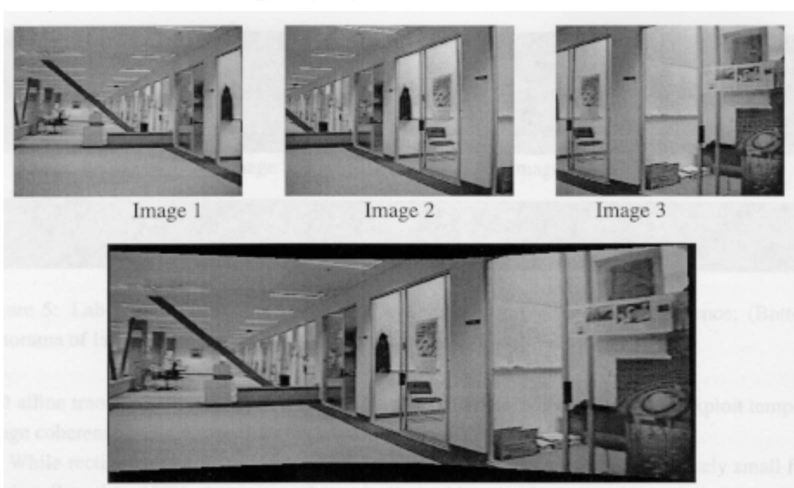
Mosaicing (II)



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Application: Detection

- Could you use correlation to find…
  - – Human faces?
  - – heads?
  - – Hands?
  - – bodies?
- What (if anything) might lead to failures?

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