PCA to Eigenfaces

CS 510
Lecture #16
February 28, 2018
A 9 dimensional PCA example

- Class 1 is dark around the edges and bright in the middle.
- Class 2 is light with dark vertical bars.
- Class 3 is light with dark horizontal bars.
- All classes initially use 2 for low value, 7 for high value.
- Each instance is corrupted by sigma=1 Gaussian Noise.

Class 1

Class 2

Class 3
Eigenspace Example 1

• Consider 3 examples from the 3 classes.

Class 1

Class 2

Class 3
The Image Matrices

• Here they are as matrices.

\[
\begin{bmatrix}
1.65 & 3.11 & 2.25 \\
3.22 & 5.79 & 3.09 \\
1.10 & 2.47 & 2.96 \\
\end{bmatrix},
\begin{bmatrix}
1.55 & 3.29 & 1.62 \\
2.91 & 3.88 & .71 \\
2.35 & 3.60 & 2.46 \\
\end{bmatrix},
\begin{bmatrix}
.80 & 2.43 & 2.04 \\
1.59 & 8.17 & .79 \\
.69 & 1.96 & 4.34 \\
\end{bmatrix},
\begin{bmatrix}
6.36 & 2.39 & 9.36 \\
6.05 & .55 & 6.60 \\
5.97 & 3.49 & 7.33 \\
\end{bmatrix},
\begin{bmatrix}
6.43 & 1.43 & 7.01 \\
7.66 & 3.20 & 6.66 \\
6.96 & 1.82 & 7.52 \\
\end{bmatrix},
\begin{bmatrix}
6.52 & .89 & 7.74 \\
4.80 & 1.97 & 7.58 \\
5.75 & 1.06 & 7.24 \\
\end{bmatrix},
\begin{bmatrix}
8.11 & 8.94 & 5.85 \\
2.63 & 2.60 & 5.16 \\
7.20 & 6.09 & 6.12 \\
\end{bmatrix},
\begin{bmatrix}
6.94 & 6.68 & 5.99 \\
3.63 & 3.15 & 1.37 \\
8.50 & 6.89 & 6.49 \\
\end{bmatrix},
\begin{bmatrix}
7.02 & 7.73 & 7.08 \\
2.75 & 2.10 & 1.91 \\
5.92 & 6.85 & 7.16 \\
\end{bmatrix}
\]
Normalized Image Vectors

• Each as a 9x1 vector, an unrolled image.
• Each has zero mean and unit length.

\[
X = \begin{bmatrix}
-0.133 & -0.117 & -0.231 & 0.0480 & 0.0530 & 0.0840 & 0.126 & 0.0810 & 0.0900 \\
0.0500 & 0.127 & -0.0450 & -0.149 & -0.202 & -0.229 & 0.197 & 0.0930 & 0.157 \\
-0.102 & -0.141 & -0.143 & 0.184 & 0.0520 & 0.124 & -0.0290 & -0.0060 & 0.0600 \\
0.0750 & 0.0930 & -0.114 & 0.0710 & 0.162 & 0.0200 & -0.129 & -0.0660 & -0.113 \\
0.324 & 0.186 & 0.505 & -0.266 & -0.116 & -0.178 & -0.157 & -0.120 & -0.177 \\
0.0910 & -0.152 & -0.163 & 0.131 & 0.135 & 0.217 & 0.0370 & -0.163 & -0.132 \\
-0.188 & -0.0140 & -0.239 & 0.0300 & 0.0860 & 0.0410 & 0.0800 & 0.172 & 0.0310 \\
0.00100 & 0.185 & -0.0710 & -0.0670 & -0.161 & -0.200 & 0.0630 & 0.124 & 0.126 \\
-0.0630 & -0.0740 & 0.0450 & 0.0320 & 0.0440 & 0.0570 & -0.0520 & -0.0150 & 0.0270 \\
\end{bmatrix}
\]
Singular Value Decomposition

- Actual values for this example.
- The Eigenvalues are 1.2, 0.55, 0.19 etc.
- The Eigenvectors are columns of U matrix.

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<table>
<thead>
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</tr>
</thead>
<tbody>
<tr>
<td>-0.29</td>
<td>-0.08</td>
<td>0.31</td>
<td>0.40</td>
<td>0.30</td>
<td>-0.17</td>
<td>-0.15</td>
<td>-0.51</td>
<td>1.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-0.14</td>
<td>-0.46</td>
<td>-0.10</td>
<td>-0.48</td>
<td>-0.27</td>
<td>-0.17</td>
<td>-0.46</td>
<td>-0.30</td>
<td>0.35</td>
<td>0.55</td>
<td>0</td>
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<tr>
<td>-0.23</td>
<td>-0.060</td>
<td>0.34</td>
<td>0.050</td>
<td>-0.49</td>
<td>-0.29</td>
<td>-0.18</td>
<td>-0.16</td>
<td>-0.66</td>
<td>0</td>
<td>0.19</td>
<td>0</td>
</tr>
<tr>
<td>-0.23</td>
<td>-0.52</td>
<td>0.64</td>
<td>0.060</td>
<td>0.12</td>
<td>0.16</td>
<td>0.050</td>
<td>-0.44</td>
<td>0.17</td>
<td>0</td>
<td>0</td>
<td>0.12</td>
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<tr>
<td>0.84</td>
<td>0.040</td>
<td>0.16</td>
<td>-0.10</td>
<td>-0.090</td>
<td>0.21</td>
<td>-0.12</td>
<td>-0.32</td>
<td>-0.30</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.080</td>
<td>-0.47</td>
<td>-0.42</td>
<td>-0.36</td>
<td>-0.45</td>
<td>-0.29</td>
<td>0.35</td>
<td>-0.22</td>
<td>0.080</td>
<td>0</td>
<td>0</td>
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<td>-0.14</td>
<td>-0.10</td>
<td>-0.38</td>
<td>0.60</td>
<td>-0.080</td>
<td>-0.020</td>
<td>-0.40</td>
<td>-0.54</td>
<td>-0.050</td>
<td>0</td>
<td>0</td>
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<td>0.080</td>
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<td>0.62</td>
<td>-0.47</td>
<td>-0.23</td>
<td>0</td>
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<td>-0.16</td>
<td>-0.050</td>
<td>0.050</td>
<td>-0.53</td>
<td>0.77</td>
<td>0.21</td>
<td>0</td>
<td>0.040</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ U \]

\[ D \]

\[ U^T \]
Subspace Projection Pictures

Legend
Class 1
Class 2
Class 3
Eigenspace Example 2

• Consider 12 4x4 images.

- Low value is 2, high is 7, noise sigma 1.0
Example 2 Subspace 3D

Legend
Class 1
Class 2
Class 3
Eigen Space Example 3

Class 1

Class 2

Class 3
Example 3 Subspace

Legend
Class 1
Class 2
Class 3
Example 3 Subspace: 99 Samples

Legend
Class 1
Class 2
Class 3
Example 3: Dimensions 3, 4 & 5

Legend

Class 1
Class 2
Class 3
Example 3 Observations

- The first two Principle Components carry no information with respect to image class.
- However, Principle Components 3 and 4 carry all the information necessary for a nearest neighbors classifier.

$$\begin{bmatrix} 4.775, 3.846, .4245, .3970, .07430 \end{bmatrix}$$

1 2 3 4 5

The Eigenvalues, which record variance along each axis, show higher PC’s have more variance:
On to Faces - Preprocessing

- Integer to float conversion
  - Converts 256 gray levels to single-floats
- Geometric Normalization
  - Aligns human chosen eye coordinates
- Masking
  - Crop with elliptical mask leaving only face visible.
- Histogram Equalization
  - Histogram equalizes unmasked pixels: 256 levels.
- Pixel normalization
  - Shift and scale pixel values so mean pixel value is zero and standard deviation over all pixels is one.
Standard Eigenfaces

Training

Training images

Eigenspace

Testing

Distance Matrix

PCA space projection
Linear Discriminant Analysis

[Google search results for geometry pca lda]

- Scholarly articles for geometry pca lda
- Tensor subspace analysis - He - Cited by 301
  The CSU face identification evaluation system: its … - Bolme - Cited by 225
  … based on local gabor filter bank and pca plus lda - Deng - Cited by 137

- Face Recognition Using Both Geometric Features and PCA ...
  ieeexplore.ieee.org/…/abs… - Institute of Electrical and Electronics Engineers - by YJ Song - 2007 - Cited by 6 - Related articles
  Face Recognition Using Both Geometric Features and PCA/LDA … the facial geometric feature and principle component analysis/linear discriminant analysis …

- The Geometry of LDA and PCA Classifiers Illustrated with ...
  www.cs.colostate.edu/…/csuldareport/report011… - Colorado State University - by JR Beveridge - Cited by 13 - Related articles
  May 30, 2001 - fixing our attention on Gaussian points clouds is that it allows us to develop an intuition for the geometry underlying both PCA and LDA spaces.
The PCA-LDA Variant

Training images are projected into Eigenspace, followed by projection into the Combined space (PCA+LDA).

Testing images are then projected into the PCA+LDA space projection, and their distances are calculated to identify similarities.
More Recently

Local Region PCA Algorithm

- 13 Local Features + Hole Face
- Self Quotient - Lighting Removal
- PCA based whitening

- 250 basis vectors per feature.
- 3500 total basis vectors.
- Fisher Criterion Weighting
- All features combined
- Similarity based upon Correlation

Self Quotient Preprocessing

Local Features
LRPCA generates 3500 Features
Summary

- PCA can be applied to any set of registered images
- It extracts the dimensions of maximum co-variance
  - In some sense, the “structure” of the domain
- Dimensions of co-variance may or may not be related to classification
- No longer competitive by itself, but still a common part of classification strategies in high-dimensional data.