Reliability of Multi-component Systems

Yashwant K. Malaiya

Reliability of Multi-component Systems

- Software system: number of modules.
- Individual modules developed and tested differently: different defect densities and failure rates.
  - **Sequential execution**
  - **Concurrent execution**
  - **N-version systems**

Sequential execution

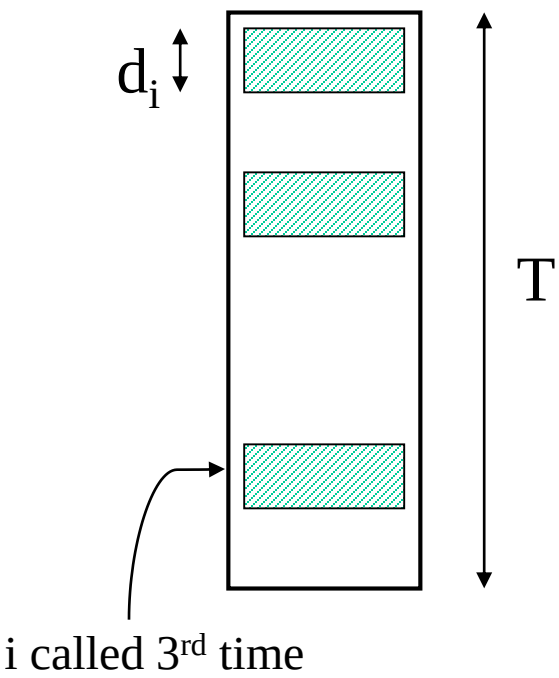
- Assume one module executed at a time.
- f_i: fraction of time module i under execution; λ_i its failure rate
- Mean system failure rate:

$$\lambda_{sys} = \sum_{i=1}^{n} f_i \lambda_i$$

Sequential Execution (cont.)

- T: mean duration of a single transaction
- module i is called e_i times during T, each time executed for duration d_i

$$f_i = \frac{e_i \ d_i}{T}$$



Sequential Execution (cont.)

- System reliability $R_{sys} = \exp(-\lambda_{sys} T)$

$$R_{sys} = \exp\left(-\sum_{i=1}^{n} e_i \, d_i \, \lambda_i\right)$$

$$\lambda_{sys} = \sum_{i=1}^{n} f_i \, \lambda_i$$

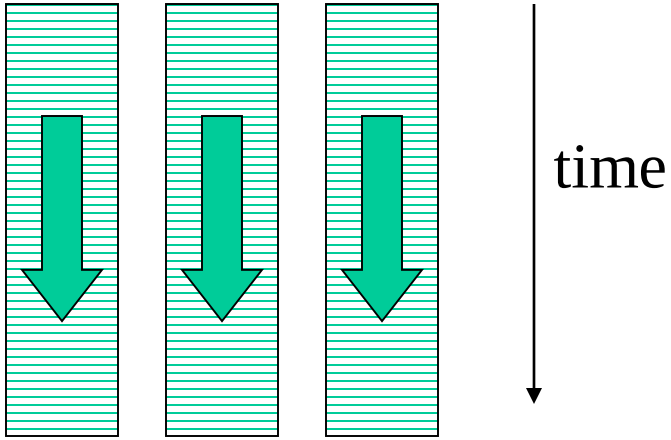
- Since $\exp(-d_i\lambda_i)$ is R_i,

$$R_{sys} = \prod_{i=1}^{n} (R_i)^{e_i}$$

$$f_i = \frac{e_i \, d_i}{T}$$

Concurrent execution

- Concurrently executing modules: all run without failures for system to run
- j concurrently executing modules

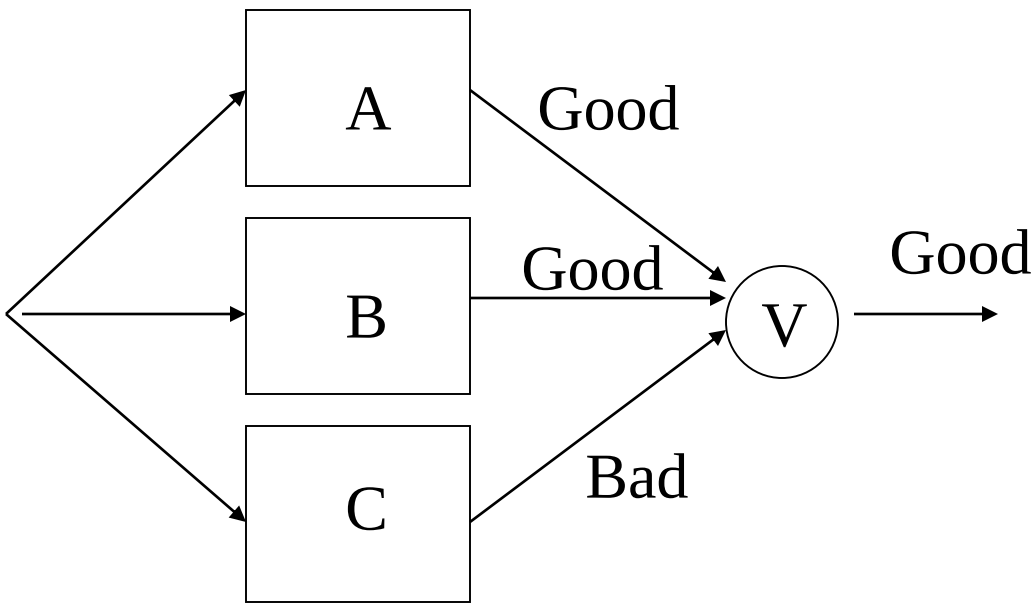


$$\lambda_{sys} = \sum_{j=1}^{m} \lambda_j$$

N-version systems

- Critical applications, like defense or avionics
- Each version is implemented and tested independently
- Common implementation uses triplication and voting on the result

N-version Systems (Cont.)



$R_{sys}=1-(1-R)^3-3R(1-R)^2$

$R=0.9 \Rightarrow R_{sys}=.972$

$\bar{R}=0.1 \Rightarrow \bar{R}_{sys}=.028$

N-version systems: Correlation

- Correlation significantly degrades fault tolerance
- Significant correlation common in N-version (Knight-Leveson)
- Is it cost effective?

N-version systems: Correlation

- 3-version system
- q_3: probability of all three versions failing for the same input.
- q_2: probability that any two versions will fail together.
- Probability P_{sys} of the system *failing* for a transaction

$$P_{sys} = q_3 + 3\,q_2$$

N-version systems: Correlation

- Example: *data collected by Knight-Leveson; computations by Hatton*
- *3-version system, probability of a version failing for a transaction 0.0004*
- *in the absence of any correlated failures*

$$P_{sys} = (0.0004)^3 + 3(1 - 0.0004)(0.0004)^2$$

$$= 4.8 \times 10^{-7}$$

- Uncorrelated improvement factor of 0.0004/4.8 x 10^{-7} = 833.3

N-version systems: Correlation

$$P_{sys} = q_3 + 3q_2$$

- Uncorrelated improvement factor of $0.0004/4.8 \times 10^{-7} = 833.3$
- Correlated: $q_3 = 2.5\times10^{-7}$ and $q_2 = 2.5\times10^{-6}$
- $P_{sys} = 2.5\times10^{-7} + 3\times2.5\times10^{-6} = 7.75\times10^{-6}$
- improvement factor: $0.0004/7.75\times10^{-6}$= **51.6**
- state-of-the-art techniques can reduce defect density only by a factor of **10!**
- Thus 3-version system may be worth considering in some cases.