1. [15 points] Induction Variables (strength reduction and induction variable elimination)
   Perform strength reduction and induction variable elimination on the following code segment
   and show the resulting code.

   ```
   p = 20
   a = 0xEFED // constant value
   loop:
   if p <= 0 goto endloop
   k = p*4
   t = k + a
   x = M[t]
   print x
   p = p - 1
   goto loop
   endloop:
   // none of the variables are used after the loop
   ```
2. [20 points] SSA and Program Optimization. The goal is to perform pessimistic global value numbering followed by copy propagation when the same program is in the SSA representation.

<table>
<thead>
<tr>
<th>SSA</th>
<th>SSA after Copy Prop</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1: j = 0</td>
<td></td>
</tr>
<tr>
<td>S2: y = read()</td>
<td></td>
</tr>
<tr>
<td>S3: x = j</td>
<td></td>
</tr>
<tr>
<td>S4: z = y + j</td>
<td></td>
</tr>
<tr>
<td>S5: v = y + x</td>
<td></td>
</tr>
<tr>
<td></td>
<td>loop:</td>
</tr>
<tr>
<td>S6: if j &gt; 10 goto endloop</td>
<td></td>
</tr>
<tr>
<td>S7: z = v</td>
<td></td>
</tr>
<tr>
<td>S8: j = j + 1</td>
<td></td>
</tr>
<tr>
<td>S9: goto loop</td>
<td></td>
</tr>
<tr>
<td>S10: endloop:</td>
<td></td>
</tr>
<tr>
<td>S11: print j, v, x, y, z</td>
<td></td>
</tr>
</tbody>
</table>

(a) [5 points] Convert the program to SSA. DO NOT use Briggs or Pruned for placing phi functions, just use the basic “minimal” rule. Write the SSA representation next to the original program in the column marked “SSA”.
(b) [10 points] Perform pessimistic global value numbering on the SSA representation. Indicate value numbers for each subscripted variable, and then modify uses and cross out definitions to show the effect of the global value number optimization.

(c) [5 points] Perform copy propagation for SSA. Do the transformation on the SSA code after global value numbering has been performed. Write the final version of the SSA code in the column entitled “SSA after Copy Prop” on the previous page. NOTE: $\phi(v_2, v_2)$ should be considered equivalent to $\phi(v_2)$. 
3. [25 points] Data dependence analysis and synchronization-free affine partitioning

   for (i=1; i<=4; i++) {
      for (j=1; j<=4; j++) {
      }
   }

(a) [5 points] For the above program, set up the data dependence problem for calculating the direction vectors (show the problem setup). Let statement 1 be in iteration space \((i, j)\) and statement 2 be in iteration space \((i', j')\). What are the direction vector representations for the data dependences?

(b) [5 points] Draw the iteration space with data dependences for the above program with a black-filled dot for S1 and a white-filled dot for S2.
(c) [5 points] For the above program, calculate two affine partitionings (one for S1 and one for S2) that result in synchronization-free parallelism on a one-dimensional array of processors. The affine mappings for the two statements are shown below. In your answer, let $C_{11} = 1$ and $c_1 = -1$ and solve for $C_{12}, C_{21}, C_{22},$ and $c_2$ using the constraints induced by the data dependences.

\[
p = \begin{bmatrix} C_{11} & C_{12} \end{bmatrix} \begin{bmatrix} i \\ j \end{bmatrix} + \begin{bmatrix} c_1 \end{bmatrix}
\]

\[
p = \begin{bmatrix} C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} i' \\ j' \end{bmatrix} + \begin{bmatrix} c_2 \end{bmatrix}
\]
(d) [10 points] Determine the range of $p$ for statements 1 and 2 (show your work). Write a loop that iterates over all possible processors and has each processor execute only the points that are mapped to it. You can leave in extra iterations and inner loop checks.
4. [10 points] Alias/Pointer Analysis (flow sensitivity)

\[
\text{main() \{ \\
    \text{int **b, *c, *d, *r, *s;} \\
    \text{int e = 7;} \\
    \text{int f = 42;} \\
    \text{c = \&f;} \\
    \text{d = \&e;} \\
    \text{r = \&f;} \\
    \text{b = \&s;} \\
    \text{*b = d;} \\
    \text{print *r, *s;} \\
    \text{if ( g > 20 ) \{} \\
    \text{b = \&r;} \\
    \text{*b = d;} \\
    \text{print *r, *s;} \\
    \text{\}} \\
    \text{print *r, *s;} \\
\}\}
\]

For the above program, perform flow-sensitive, context-sensitive alias analysis (FSCS) and flow-insensitive, context-sensitive alias analysis (FICS). Show you work in the space provided next to the program.

(a) For FSCS, what is the points-to set for the variables \text{r} and \text{s} at each of the print statements? Based on these points-to sets, what can the compiler determine about the output of the program?

(b) For FICS, what is the points-to set for the variables \text{r} and \text{s} at each of the print statements? Based on these points-to sets, what can the compiler determine about the output of the program?
5. [25 points] Interprocedural Analysis (context and flow sensitivity)

```c
int g, h, *p, *r;

void goo(int *a, int *b) {
    *b = 60;
}

void foo(int *x, int *y) {
    *x = 21;
    goo(x, y);
}

int main() {
    g = 2;
    h = 77;
    p = &g;
    r = &h;

    foo(p, r);
    printf("*p = %d, *r = %d\n", *p, *r);

    foo(p, p);
    printf("*p = %d, *r = %d\n", *p, *r);
}
```

For the above program answer the following questions:

(a) What will the output of the program be?
(b) Construct the interprocedural control-flow graph for the program, treating all calls as control flow and cloning each procedure that is called at more than one callsite in the program text.

(c) What does a data-flow analysis that keeps track of points-to sets and reaching constants information determine about the output of the program?
(d) Now draw a graph where each procedure is cloned for all possible call paths to that procedure (this is possible because the given program does not involve recursion).

(e) Now what does a data-flow analysis that keeps track of points-to sets and reaching constants information determine about the output of the program when using the fully context-sensitive interprocedural control-flow graph?
6. [5 points] Where could a representation for loops (polyhedral or presburger sets) fit into the MiniJava compiler? Support your answer by describing what analyses need to be done before representing loops and what transformations and translations need to be done after the loops have been manipulated. (Keep your answer in the 5-10 sentences range).
7. [5 points extra credit] Suggest a project for next year’s CS 553 that is different from the projects you were assigned or asked about in the homeworks.

(a) Describe the project in one paragraph.
(b) What concept will the students learn in more depth by doing the project?
(c) What experimental results should be presented in the project report?