CS 560: Homework 2

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Consider the following System of Recurrence Equations, where $\mathcal{D}_X = \{i,j \mid 1 \le (i+j) \le i \le n\}$ and $\mathcal{D}_Y = \{i,j \mid -i \le j < 0; 2 \le i \le n\}$ that returns an output b defined over $\mathcal{D}_b = \{i \mid 1 \le i \le n\}$.

$$b_i = X[n, i-n] \tag{1}$$

$$Y[i,j] = \begin{cases} i+j=0 & : \quad a_i \\ i+j>0 & : \quad f_Y(X[i-1,j+1], Y[i,j-1]) \end{cases} \tag{2}$$

$$X[i,j] = \begin{cases} i=1; j=0 & : \quad a_i \\ i>1; j=0 & : \quad Y[i,j-1] \\ j<0 & : \quad f_X(X[i-1,j+1], Y[i,j-1]) \end{cases} \tag{3}$$

Problem 1: Classify this according to the taxonomy described in the notes. [2 pts]

Problem 2: Draw a neat diagram (no freehand sketches please) of the domains of the two variables (for $n = 10$). Please make sure that they use the same coordinate system (i.e., they are superimposed on each othere wherever they overlap), and that you use different colors (or other conventions like bullets vs boxes) for each of the two variables. Show the dependences in the different regions (don't clutter up the figure by showing the dependencies for *every* point, but pick a few representative ones). [10 pts]

Problem 3: Write a simple program (in `C` or `Java`) that implements this SRE. Assume that a is an input array, and make sure that your program has no more than a doubly nested loop over the i and j indices (i.e., it is *not* memoized). To test your program assume that $f_Y(x,y)$ is min and $f_X(x,y)$ is max. [10 pts]

Problem 4: Write a `C` or `Java` program to implement the following SRE that defines $m[i,j]$ over a triangular domain $\mathcal{D}_m = \{i,j \mid 1 \le i \le j \le n\}$:

$$m[i,j] = \begin{cases} i=j & : \quad 0 \\ i<j & : \quad \min\limits_{i \le k < j} (m[i,k] + m[k+1,j] + p_{i-1} p_k p_j) \end{cases} \tag{4}$$

where p_i for $i = 0 \ldots n$ is an input. Your program may or may not be memoized (the memoized version is easier to write). [15 pts]

Describe briefly the decisions you would have to make (or have already made) to write a non-memoized version of this program. [3 pts]

Problem 5: You are given a 1-D array of real values, a_i, for $i = 1 \ldots n$. We are interested in computing a function $M(i,j)$ that returns the largest value of a in the interval $\langle i,j \rangle$. Develop (explain your reasoning) an equation that specifies this function. Its complexity should be no more than $\Theta(n^3)$. [10 pts]

For (some) extra credit, describe an equation (or an SRE) that computes the answer with $\Theta(n^2)$ complexity. [10 pts]

For (considerable) extra credit, *systematically derive* the $\Theta(n^2)$ solution. [10 pts]