CS 560 Spring 2008: Take-home Mideteremm

due midnight Friday, March 14

Problem 1: Why Equations: [25 pts]

Derive, using a logic similar to that used in the lecture notes, the equations for LU factorization (without pivoting) of a matrix in Hessenberg[1] form. [15 pts]

Draw the domains of the vaiables. [5 pts]

What is the complexity of the program? Hint: it can be deduced from the volumes of (i.e., the number of integer points in) the domains. [5 pts]

Problem 2: Systolic Synthesis: [40 pts]

Derive a systolic array that computes a $2m$-point convolution, $y_i = \sum_{j=0}^{2m-1} w_j x_{i-j}$ for the special case when the weights are symmetric, i.e., $w_j = w_{2m-j}$. Assume that the boundary conditions are as in the lecture notes (Chapter on Advanced Systolic Design).

The key constraint is that multipliers are expensive, so you should exploit symmetry and use only m multipliers (i.e., your architecture should have m PEs, each with a single multiplier). Describe the systematic design of your architecture, including:

- mathematical "preprocessing" of the equations;
- serialization (replacing reductions by accumulations);
- localization (replacing long dependences by propagations);
- scheduling;
- allocation;
- constructing the CoB and transforming the equations; and
- description of the final hardware.

Problem 3: Implementing Equations: [15+20=35 pts]

Consider the following SARE that computes a lower triangular matrix, L, given another input (triangular) matrix A.

$$L_{i,j} = \begin{cases} j < i : \dfrac{1}{L_{j,j}} \left(a_{i,j} - \sum_{k=1}^{j-1} L_{i,k} L_{j,k} \right) \\ j = i : g\left(a_{i,i} - \sum_{k=1}^{i-1} L_{i,k}^2 \right) \end{cases} \tag{1}$$

You are to write two programs to evaluate this equations. The first is a demand driven, memoized program, and the second is a scheduled, and efficient one that uses the smallest amount of memory (within a constant factor). Analyze the running time and space complexity of both programs.

[1] A matrix is said to be in Hessenberg form if all its entries for $i \geq j+2$ are zero. It is thus "nearly" uppper triangular, but there is one diagonal below the principal.